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Targeted Advertising with Vertically Differentiated Products

Lola Esteban and José M. Hernández

Department of Economic Analysis

University of Zaragoza

Abstract: This paper presents an incomplete information game of pricing and targeted advertising with vertically differentiated products. Firms have incomplete information about production costs and can inform consumers about price and product characteristics by either using the mass media, which reaches the whole market, or the specialized media, which reaches only the most eager consumers. In this framework, we show that pure strategy Bayesian Nash equilibria exist, and that, as compared to mass advertising, targeting leads to higher prices. Further, we offer an explanation for permanent market fragmentation, which is related to high production costs, and prove that the probability of such fragmentation is directly related to the degree of precision of the targeting technology. Finally, we show that targeting can only occasionally reduce welfare.

Keywords: informative advertising, targeting, vertical differentiation, Bayesian games, price discrimination.

JEL Classification: D43; D82.

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Correspondence: José M. Hernández. Departamento de Análisis Económico, Facultad de Económicas, Gran Vía 2, 50.005 Zaragoza, Spain, Phone: 34 76 761823, Fax: 34 76 761996, e-mail: hernande@unizar.es.

1 Introduction

The way in which firms promote their products has undergone significant change during recent years. Traditionally, sellers have used the mass media to reach their potential customers, and marketing managers have been concerned about how to reduce to the greater extent possible the waste of ads, that is to say, the exposure to the advertising campaign of those consumers who are not interested in their products. However, as the result of both the progressive fragmentation of traditional media (radio, TV, etc.) and the proliferation of new and highly specialized advertising channels (the Internet, cable TV, etc.), firms have nowadays the possibility to target their ads on particular segments of the potential demand, thus improving advertising cost efficiency. Against this background, the aim of this paper is to investigate the strategic use of targeted advertising, in such a way that we can improve our understanding about how the move from traditional mass advertising to targeting can affect both market outcomes and welfare.

We formulate a model of price competition in which two firms use informative advertising to promote sales. Consumers are unaware of the existence of the goods, and sellers can inform them about the existence, price or product specifications by using either mass advertising, which reaches the whole potential market, or specialized advertising, which targets the ads on a particular segment of the market. In this framework, the fundamental issue is how firms can target the ads, i.e. how the targeting technology relates to product's demand. In this regard, it is important to note that the degree of media specialization is often positively correlated with consumers' valuation of the good, in such a way that firms can frequently target their ads *only* on the most eager consumers.¹ In order to accommodate this type of targeting into a price competition model, it is necessary to impose a particular structure on consumers' preference ordering. In particular, both firms can be in accordance about who are the "*most eager customers*" only if all consumers agree over the preference ordering, that is to say, if when products are offered at the same price, all customers choose to purchase the same one. Accordingly, we think that a natural way to study the effects of targeted advertising in a price competition model is in the context of vertically differentiated products, in the spirit of Shaked and Sutton (1982, 1983). Therefore, we consider that each firm supplies a good with a different level of quality and, in this context, targeting will mean that there exists a given specialized advertising media which allows firms to concentrate their ads *only* on a subset of those consumers who value quality most.

Our first goal is to describe the set of Nash equilibria of a *complete information game* in which both firms decide simultaneously their pricing and advertising strategies, for a given level of product

¹Esteban, Gil and Hernández (2001) and Esteban, Hernández and Moraga (2001) report this evidence and offer examples about this class of targeting technologies.

qualities.² We begin by describing the equilibrium pricing strategy when firms can use only the mass media, and both sellers compete for the marginal consumer. This solution, which equals the full information outcome, constitutes a reasonable benchmark against which we can compute the impact of targeting on market performance. Next, we show that the high-quality firm has a greater incentive to target the advertising, given that the use of specialized media could allow this firm to reach its potential customers at a lower advertising cost. Obviously, the target of the specialized media does not necessarily have to coincide with the segment of the market served by the high-quality firm and, therefore, we analyze various targeting technologies characterized by different levels of targeting precision. In fact, we find that the impact of targeting on the market outcome depends crucially on the level of targeting precision. In particular, if this level is *low*, that is to say, if the specialized media reaches only a relatively large subset of high valuation consumers, then there exist two types of pure strategy Nash equilibria in which the high-quality firm targets the ads and both firms compete for fully informed consumers. In the first of these equilibria, the low-quality firm uses mass advertising and, as compared to the benchmark, targeting has no effect on market prices. In the second, both firms target their ads, which reduces the degree of consumers heterogeneity, thereby stimulating competition and lowering market prices.

We must note that these two basic equilibria exist only under quite restrictive market conditions, namely, low levels of targeting precision. In order to understand this result, we next study how highly precise targeting affects the pattern of price competition between firms. The key point here is that when a firm targets its ads, the rival can use mass advertising to reach uninformed consumers who are not in the target set of the competitor, thus obtaining a captive market. Therefore, targeting can generate a new source of product differentiation: information. It is obvious that firms might have a high incentive to monopolize such a captive market, and so the central question in the analysis of targeting is whether this type of advertising can indeed lead to an equilibrium with market fragmentation, that is to say, an equilibrium with a local monopoly. Our complete information model turns out to be inappropriate to answer this question, given that, as we demonstrate, when targeting is highly precise a Nash equilibrium of the pricing-targeted advertising game does not exist.

On this basis, the second, and main goal of this paper, is to formulate an extended model of targeted advertising which makes it possible to predict the market outcome when the level of targeting precision is high, in such a way that we can determine whether targeting can in fact lead to market fragmentation. To that end, we note that market competition very often takes place

²In most of the paper we consider that the levels of quality are given. However, we will also provide a brief discussion about the relationship between targeted advertising and the optimal supply of quality.

in a context of *incomplete information*. Accordingly, we propose a strategic game of pricing and targeted advertising in which the cost of the low-quality firm is private information whereas, for the sake of simplicity, we assume that the cost of its rival is public information. In the basic version of our incomplete information model, we consider that the low-quality firm's cost can *a priori* take two values, low or high, and that its type is drawn from a probability distribution which is common knowledge. In this setting, we demonstrate that, for relatively high levels of targeting precision, there exists a pure strategy Bayesian Nash equilibrium which is consistent with market fragmentation and the creation of a local monopoly. More precisely, the market is fragmented when the low-quality good is produced at a high cost, whereas if the production cost is low, then both firms compete for fully informed consumers. A particularly noteworthy result is that, regardless of whether the market is fragmented or not, and as compared to mass advertising, targeting always leads to higher market prices, and so this advertising strategy allows both firms to achieve higher profits. Furthermore, we find that the extent to which targeted advertising allows firms to raise their prices is inversely related to the level of precision of the targeting technology. Finally, we show that market fragmentation can induce a welfare loss due to the typical quantity distortion introduced by a monopolist. Therefore, as compared to mass advertising, targeting can generate a trade-off between higher advertising cost efficiency and greater monopoly power. We compute the model in order to shed light on the relative strength of these forces, and find that although targeted advertising might have a detrimental effect on welfare, this possibility has very limited practical relevance.

In a first extension of our work, we set-out to endogenize the probability that targeting results in market fragmentation, so as to study how this probability relates to market conditions. We achieve this goal by considering that the low-quality firm's cost follows a continuous probability distribution. On the basis of this extended model, we show that the probability of monopolization increases if: (i) the cost of the high quality firm decreases, (ii) the degree of targeting precision increases and, finally, (iii) if consumers' valuation of the good increases.

In a second extension of our work, we explore the relationship between targeted advertising and targeted prices. In particular, we show that when targeting is highly precise and information is *complete*, the possibility of price discrimination makes it possible to find a *pure strategy* Nash equilibrium of the price and targeted advertising game, which is consistent with market fragmentation. The key point here is that the low-quality firm can monopolize a segment of the market and, simultaneously, capture a segment of its rival's "natural" market by price discriminating with discount coupons. Obviously, this targeting strategy benefits the low-quality firm to a great extent, whereas the impact on the high-quality firm's profits depends on whether the gain in advertising cost

efficiency induced by targeting compensates for the loss of revenues due to the higher competitive pressure induced by the use of coupons. We have computed the model to determine which of these effects dominates, and find that, as compared to mass advertising, targeting tends to reduce the profits achieved by the high-quality firm.

The analysis of targeted advertising on particular segments of the potential demand has received only limited consideration in the informative advertising literature.³ In monopolized markets, Esteban, Gil and Hernández (2001) build on the “*constant-reach independent-readership*” advertising technology formulated by Grossman and Shapiro (1984) to propose a targeting advertising technology based on “*specialized magazines with nested readerships*,” which allows a monopolist to target the ads on those consumers with a higher valuation of the good. In this framework, they show that targeting leads to a higher market price and thus, is potentially detrimental from a welfare perspective.⁴ In a strategic context, Bester and Petrakis (1996) consider the effect of advertising targeted on low valuation consumers. They show that this type of targeting leads firms to engage in price discrimination by means of discount coupons, which stimulates competition in the market. To our knowledge, the only works which analyze targeted advertising on high valuation consumers in a strategic context are those of Roy (2001), Galeotti and Moraga (2003), and Iyer, Soberman and Villas-Boas (2003). The first of these examines targeting in a two-stage model, where two firms decide, first, the advertising strategy and then compete in prices. The timing of this game suggests that advertising has a long-run nature, perhaps to create brand loyalty and, later, consumers learn prices without cost. This special nature of targeted advertising allows firms to commit themselves not to invade the rival’s market, in such a way that this becomes fragmented and both firms enjoy a monopolistic position in their local markets.

More closely related to our work are Iyer *et. al.* (2003) and Galeotti and Moraga (2003), who study targeting when firms inform about prices and, therefore, in an environment where advertising has a short-run nature. In particular, Iyer *et. al.* (2003) examines a market where two firms compete

³This literature has distinguished between advertising that is *directly* informative, i.e., that conveys ‘hard’ information, e.g. existence, price, location or technical features, (see e.g. Bester, 1995; Bester and Petrakis, 1995; Butters, 1977; Caminal, 1996; Grossman and Shapiro, 1984; Moraga-González, 2000; Robert and Stahl, 1993; Shapiro, 1980; Stahl, 1994 and Stegeman, 1991), and advertising that is *indirectly* informative, i.e. that functions as a signal of private information, e.g. quality (see e.g. Bagwell, 1988; Bagwell and Ramey, 1990; Kihlstrom and Riordan, 1984; Milgrom and Roberts 1986). Our paper deals with informative advertising that is directly informative and thus relates to the first strand of this literature.

⁴Esteban, Hernández and Moraga (2001) adopting the same framework, study the relationship between targeting and product quality. They find that targeted advertising has a bearing on both the price and the design of new products. Targeted advertising is also analyzed in Esteban, Gil and Hernández (2004), who investigate the effects of database direct advertising.

with horizontally differentiated products. They assume that each firm has a “loyal” segment of the market, but where they compete for the remaining consumers, who are price-sensitive. Further, each firm can either inform the whole market, or target the ads on one of the market segments. In this framework, the authors show that an equilibrium must involve random pricing, and so their model explains price dispersion on the basis of both mass and targeted advertising. In addition, with mass advertising firms achieve zero profits, whereas targeting leads to a mixed strategy equilibrium with positive profits. Regarding the advertising strategy, they show that sellers advertise the price to their loyal segment with probability one, but advertise to comparison shoppers from time to time. As a result, only *temporary market fragmentation* can arise, in the sense that sporadically both firms offer their product exclusively to their loyal customers. Galeotti and Moraga (2003) examine targeted advertising in a homogeneous good market that is segmented only in regard to the advertising media through which potential consumers can be reached by the firms. In this framework, they show that equilibria must exhibit both random pricing and random advertising, in such a way that the possibility of *permanent* market fragmentation is also discarded. However the price-dispersed equilibrium offers an explanation for temporary market fragmentation based not on loyalty, but rather on purely strategic considerations. Thus, their model illustrates the role of targeted advertising in generating positive profits in highly competitive markets.

We contribute to improve the understanding about how targeted advertising can affect market outcomes in several respects. First, our work builds on the targeting technology proposed by Esteban, Gil and Hernández (2001), which allow firms to concentrate the ads only on the most eager consumers. This type of targeting is frequently used by firms, and so we examine strategic targeted advertising under a quite common type of market segmentation. In this framework, our incomplete information model of price competition proves that targeting is likely to lead to higher prices. Secondly, we are able to offer an explanation for *permanent* market fragmentation and, further, to analyze which market conditions determine the probability of such fragmentation. In addition, our model allows for a flexible targeting technology, in the sense that the specialized advertising media can reach the potential market with different degrees of precision. This enables us to understand how the current increasing precision of targeting technologies can affect the market outcomes. Thirdly, we provide a welfare comparison between mass advertising and targeting, and discuss the extent to which targeting might have a detrimental effect on social welfare. Therefore, our work will also reveal whether the results reported by Esteban, Gil and Hernández (2001) apply to a competitive environment. Finally, we note that our set-up combines both informational and vertical differentiation which, to our knowledge, is novel in the industrial organization literature.

The remainder of the work is organized as follows. Section 2 builds the foundations of the

pricing-targeted advertising model and provides some preliminary results. Section 3 analyzes targeted advertising in an incomplete information framework. In Section 4, we extend the analysis of targeting in two directions: on the one hand, we consider a continuous distribution of production costs and, on the other, we explore the relationship between targeted advertising and targeted prices. Finally, Section 5 closes the paper with some concluding remarks. All the proofs are relegated to an Appendix.

2 The Basic Model: Preliminary Results

We consider a market with a unitary mass of consumers who demand, at most, one unit of a product. A consumer's utility is $U = v + \theta s - p$, when he buys a good of quality s at a price p , and 0 if he does not buy. The parameter $v > 0$ represents consumers' common valuation of the product, independent of the level of quality. The parameter θ of taste for quality is uniformly distributed across the population of consumers in the interval $[a, b]$, with $b - a = 1$, in such a way that consumers can be indexed by the value of their taste parameter. On the supply side of the market, there are two firms $i = 1, 2$, which produce two goods of a given quality $s_i > 0$, with $s_2 > s_1$. These firms have constant-returns-to-scale technologies, and the marginal cost of production depends on the level of quality $C_i(s_i) = c_i s_i$, where c_i can be interpreted as firm i 's level of cost efficiency. Further, the production of a higher quality is more costly, i.e. $C_2(s_2) > C_1(s_1)$.

Consumers ignore the existence, the quality and the price of the goods, and so a potential consumer cannot buy the product unless sellers invest in advertising.⁵ In order to inform consumers, the firms can use either the mass media, which spread the ads on the entire population of potential buyers $[a, b]$, or the specialized media, which target the advertising on a given subset of these consumers. More precisely, and according to the targeting technology proposed by Esteban, Gil and Hernández (2001), we assume that sellers can target the campaign only on the most eager consumers.⁶ In terms of our model, this definition of targeting implies that there exists a specialized

⁵With this assumption, we are in line with that stream of the informative advertising literature which considers that consumers search cost is high relative to the expected surplus offered by the goods, in such a way that potential buyers are passive and, in the absence of information, do not purchase any good (see, for example, Grossman and Shapiro, 1984; Stegeman, 1991; Stahl, 1994).

⁶One example of this targeting technology is the case of "*specialized magazines with nested readerships*." Thus, there are magazines containing general information on sports, medicine, computers, family matters, etc., as compared to those specialized in particular sports (soccer, basketball, golf, etc.), medical specialties (surgery, radiology, dermatology, etc.), computer issues (video-games, Internet, etc.) or leisure activities (fitness, decoration, gardening, etc.) which reach high valuation consumers. Accordingly, firms can target the most eager consumers by using the specialized media. However, in order to reach low-valuation consumers, sellers can only use general magazines which

advertising media which reaches those consumers in the interval $[z, b]$, with $b > z > a$. Thus, if t denotes the target of the advertising campaign, we assume that firm i can insert the ads either in the mass media, $t_i = a$, or in the specialized media $t_i = z > a$, and that the value of z is exogenous.⁷ We further consider that when a firm advertises the product in a segment of the market, all consumers in that segment become informed about the existence, price and characteristics of the good. Advertising is costly, and the cost of a campaign depends on the size of the target market. If A_0 denotes the cost of informing all consumers in $[a, b]$, and $A_1 = A_1(z)$ denotes the cost of a campaign targeted on $[z, b]$ with $z > a$, then, given that targeted advertising reduces the number of consumers reached by the campaign, it seems natural to assume⁸ that $A'_1(z) < 0$, i.e. $A_0 > A_1$ for all $b > z > a$.

We are now in a position to analyze the equilibrium of the simultaneous move game in which both firms decide their pricing-targeted advertising strategies. In equilibrium, firm i takes as given its rival's price-advertising strategy, and decides the strategy (t_i, p_i) , that is to say, the type of advertising campaign, mass $t_i = a$ or targeted $t_i = z$, and the price of the product with quality s_i , so as to maximize profits. For future reference, let (p_1^m, p_2^m) denote the unique equilibrium price strategies when both firms use mass advertising and compete for the fully informed marginal consumer, θ^m , where $\theta^m = \frac{p_2^m - p_1^m}{\Delta s}$, with $b > \theta^m > a$ and $\Delta s = s_2 - s_1$ (see the Appendix for details on this equilibrium). Starting from this equilibrium, we can first note that if z is sufficiently high, $z > \theta^m$, both firms are likely to have a low incentive to target their campaigns, given that firm 1 could not reach any of its potential customers, whereas firm 2 could reach only a fraction of its potential demand. Therefore, we begin by focusing our analysis on the most interesting case namely, that in which $z \leq \theta^m$. Under this condition, we first prove that it does not exist a Nash equilibrium of the price-targeted advertising game in which both firms use mass advertising.

Lemma 1 *If $z \leq \theta^m$, the advertising strategy $(t_1 = a, t_2 = a)$ cannot be part of an equilibrium.*

Lemma 1 simply suggests that firms have a high incentive to target their advertising campaigns in order to improve advertising cost efficiency. In particular, we note that, when $z \leq \theta^m$, targeting is particularly attractive for firm 2 (the high-quality firm), given that it allows this firm to better focus the campaign on its set of potential customers, i.e. on those consumers with a high θ . In fact, the proof of Lemma 1 shows that, given $(t_1 = a, p_1^m)$, firm 2's best response is $t_2 = z$, and reach the whole potential market.

⁷The use of both mass and specialized advertising only makes sense if the firm can implement some price discrimination device. We analyze this possibility in section 4.

⁸Esteban, Gil and Hernández (2001) and Esteban, Hernández and Moraga (2001) provide empirical evidence confirming this intuition for the case of “specialized magazines with nested readerships.”

therefore, it makes sense to look for equilibria in which firms 2 targets its ads. We now characterize two simple Nash equilibria, that we denote as E1 and E2 respectively, in which $t_2 = z$. Let (p_1^t, p_2^t) denote the equilibrium prices under targeted advertising. Then:

Proposition 1 *If $z < \theta^m$, there exists a non-empty set of parameters $(a, v, z, s_1, s_2, c_1, c_2, A_0, A_1)$ for which the following pure strategy Nash equilibria of the price-targeted advertising game exist:*

- (i) $E1 = [(t_1 = a, p_1^t); (t_2 = z, p_2^t)]$ with $p_i^t = p_i^m$, $i = 1, 2$;
- (ii) $E2 = [(t_1 = z, p_1^t); (t_2 = z, p_2^t)]$, with $p_i^t < p_i^m$, $i = 1, 2$.

There are two types of targeted advertising equilibria in which both firms compete for fully informed consumers. In the first of these, only firm 2 targets the ads, which implies that those consumers with a taste parameter in the range $[a, z]$ receive information only from firm 1 (the low-quality firm) and, therefore, this segment of consumers constitutes a captive market. Under the condition $z < \theta^m$, firm 1 has two options: either to monopolize this captive market, or to compete with firm 2 for those consumers in $[z, b]$. Proposition 1 states that, under certain market conditions, firm 1 finds it optimal to compete and that, as compared to mass advertising, this targeting has no impact on market prices. The reason why prices do not change is that in this equilibrium the marginal consumer is fully informed and so, regarding prices, competition with mass or with targeted advertising is identical. Therefore, this pricing-advertising strategy results simply in higher advertising cost efficiency, and is thus welfare improving.

In the second equilibrium, both firms target their advertising campaigns on the same subset of high valuation consumers, which reduces the amount of consumer heterogeneity to $[z, b]$ and leaves the segment of the market $[a, z]$ ignorant about the existence of both goods. Under these conditions, and as compared to mass advertising, price competition becomes more intense, which leads both firms to charge lower prices. Obviously, this pricing-advertising strategy benefits those consumers who receive an ad. However, in terms of welfare, the impact of targeted advertising is ambiguous, given that the higher advertising cost efficiency achieved by firms can be overcome by the loss of surplus suffered by those consumers who buy a product under mass advertising and that now are uninformed.

In summary, Proposition 1 indicates that targeted advertising can lead to quite different market outcomes. Therefore, an interesting question is to determine the practical relevance of both equilibria, which requires that we take a closer look at the existence conditions derived in the Appendix. To that end, we can note that, essentially, targeting generates two effects: on the one hand, it increases advertising cost efficiency and, on the other, it allows firms to monopolize the segment of the market $[a, z]$. Given that the former effect is primarily related to the value of A_1 , and the latter is mainly re-

lated to the value of z , in Figure 1 we examine the set of values of (A_1, z) for which E1 and E2 exist, for a given value of the remaining parameters, $[a, v, s_1, s_2, c_1, c_2, A_0] = [0, 150, 100, 240, 0.5, 0.5, 10]$, which we will refer to as the “base-case market scenario.”⁹

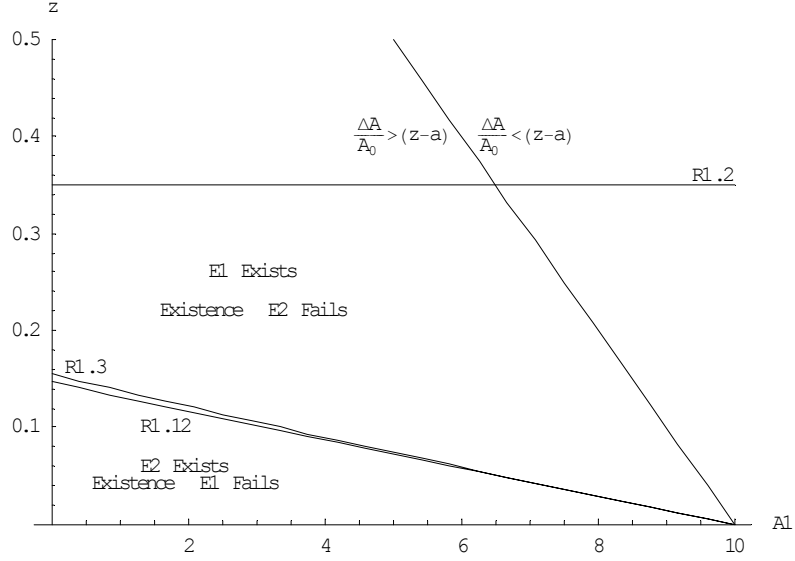


Figure 1: Existence E1 and E2

From Figure 1 it follows that equilibrium E1 exists only for intermediate values of z , i.e. for those values of z comprised between R 1.2 and R 1.3 (see Appendix). This is so because, for a high z relative to θ^m , $0.35 < z < \theta^m = 0.5$, the low-quality firm would deviate by monopolizing the segment of the market $[a, z]$, whereas for sufficiently low values of z , it would deviate by targeting the advertising campaign, $t_1 = z$. With regard E2, Figure 1 shows that this equilibrium exists only very low values of z , $z < 0.15$. More precisely, E2 exists only for those values of z below R 1.12, given that for higher values firm 2 would deviate by setting $t_2 = a$.

To gain further insight into the practical relevance of E1 and E2, we note that the cost saving induced by targeting is likely to be related to the size of the target market. Accordingly, we think that a reasonable way to represent the efficiency gain induced by targeted advertising is to assume $A_1 = A_0(1 + a - z)$, i.e. $\frac{\Delta A}{A_0} = (z - a)$, with $\Delta A = A_0 - A_1$. We have represented this condition in Figure 1, and find that whilst E1 might well satisfy $\frac{\Delta A}{A_0} \leq (z - a)$, equilibrium E2 exists only if targeting induces a percentage of cost saving substantially greater than $(z - a)$.

⁹We would argue that this market scenario can be considered reasonable because, for $t_1 = t_2 = a$, it generates an advertising cost-to-sales ratio of approximately 20%. This is consistent with the stylized fact that when a firm launches a new product, it usually incurs an advertising cost that represents an average of about 20% of sales (Kotler and Armstrong, 1998).

Therefore, we conclude that E2 has a very limited practical significance, whereas the relevance of E1 is restricted to relatively low values of z . In order to better understand the difficulty that arises when $(t_1 = a, t_2 = z)$ and $z > 0.35$, it is convenient to focus the discussion on the case in which the level of targeting precision is sufficiently high, i.e. as z approaches θ^m . Starting from E1, and given $(t_1 = a, p_1^m)$, if sellers can target their advertising campaigns, firm 2's best response will be $t_2 = z \approx \theta^m$, which causes products to be differentiated along two dimensions, quality and information, thus changing substantially the pattern of price competition between firms. In particular, given $(t_2 = z \approx \theta^m, p_2^m)$, firm 1's optimal strategy is to set $t_1 = a$ and charge the monopoly price, $p_1^M > p_1^m$, to the segment of imperfectly informed consumers, $[a, z]$, thus fragmenting the market. On this basis, the natural question to ask is whether such market fragmentation, based on the advertising strategy $(t_1 = a, t_2 = z)$, can in fact be an equilibrium. The problem with this outcome is that both products are strategic complements, and so firm 2 would respond to the monopolization of $[a, z]$ with a substantial increase in its price, $p_2 > p_2^m$. This will lead firm 1 to compete for $[z, b]$ by lowering p_1 below p_1^M which, in turn, will induce firm 2 to lower the price. Finally, given a low p_2 , firm 1's best response would be, once more, to monopolize $[a, z]$ by raising the price up to p_1^M , thus starting the same price-cycle again. The following Lemma provides a formal proof of this result.

Lemma 2 *If $z = \theta^m$, the simultaneous move pricing-targeted advertising game does not have a pure strategy Nash equilibrium in which $(t_1 = a, t_2 = z)$.*

Lemma 2 states that for a high level of targeting precision, if $t_2 = z$ it is not possible to sustain a market equilibrium in which $t_1 = a$. Further, from the discussion of Proposition 1 it follows that for a relatively high z , the advertising strategy $(t_1 = z, t_2 = z)$ will not be part of an equilibrium. Therefore, the analysis of this Section suggests that when the level of targeting precision is high, firm 2 will target the ads, and a pure strategy Nash equilibrium does not exist, which makes it difficult to predict the result of market interaction. In the light of this, and as stated earlier, the main goal of this paper is to formulate an extended model of targeted advertising which be able to predict the market outcome for relatively high values of z , so that we can determine whether targeting can indeed lead to market fragmentation. To that end, we note that market competition usually takes place in a context of incomplete information. Accordingly, in the next section we study targeted advertising under the hypothesis that firms have limited information about production costs.

3 Equilibrium under incomplete information

In this section we examine a pricing-targeted advertising game in which the cost-efficiency parameter of the high-quality firm, c_2 , is common knowledge, whereas the cost-efficiency parameter of the low-quality firm is private information. For the sake of simplicity, we begin by considering that firm 1 can be either a low-cost type, $c_1 = c_3$, or a high-cost type $c_1 = c_4 > c_3$. Firm 1's type is drawn from a simple discrete probability distribution which is common knowledge. In particular, we assume that, *ex ante*, firm 1 has a probability λ of having a low-cost and a probability $(1 - \lambda)$ of having a high-cost, with $1 > \lambda > 0$. At the beginning of the game, firm 1 privately observes the random variable's true realization, in such a way that it learns its own type; thereafter, both firms simultaneously decide their pricing-targeted advertising strategies.

The aim of this section is to investigate whether this game has a pure strategy Bayesian Nash equilibrium based on the advertising strategy $(t_1 = a, t_2 = z)$, which is consistent with market fragmentation and the creation of a local monopoly. To that end, we begin by describing the strategy space of both firms. Taking $t_2 = z$ as given, if firm 1 sets $t_1 = z$, both firms will compete for fully informed consumers in $[z, b]$ which, as we have already seen in section 2, leads to high price competition and, therefore, low profits. On the other hand, if $t_1 = a$, given the rival's price, firm 1 can choose between competing for the segment of the market $[z, b]$, or monopolizing the segment $[a, z]$ of imperfectly informed consumers. In the former case, firm 1 will serve a demand $D_1 = \left(\frac{p_2 - p_1}{\Delta s} - a\right)$, with $\frac{p_2 - p_1}{\Delta s} > z$, whereas, in the latter, firm 1 will fix the monopoly price, p_1^M , and serve a lower demand $D_1^M = \text{Min}\left[z - \frac{p_1^M - v}{s_1}, z - a\right]$. Taking into consideration that the size of the market that a firm is willing to serve is usually negatively correlated with the level of production cost, it makes sense to think that firm 1 could find it optimal to set a low price and compete for $[z, b]$ when $c_1 = c_3$, and to set p_1^M and monopolize $[a, z]$ when $c_1 = c_4$.

Given this later pricing-targeting strategy, firm 2 has to decide the reach of the advertising campaign, $t_2 = z$ or $t_2 = a$. Let us first consider that $t_2 = z$. Firm 2 does not observe c_1 , and so it will not be able to anticipate the rival's price strategy with certainty. However, it knows that with probability $(1 - \lambda)$ firm 1 does not compete for $[z, b]$, and so it will face a low competitive scenario and serve a demand: $\text{Min}\left\{b - z, b - \frac{p_2 - p_1^M}{\Delta s}\right\}$. Further, with probability λ , firm 1 sets a low price to compete for $[z, b]$, and so firm 2 will face a highly competitive scenario with a more elastic demand: $b - \frac{p_2 - p_1}{\Delta s}$, with $\frac{p_2 - p_1}{\Delta s} > z$. This implies that firm 2 must choose p_2 by taking account of the marginal effect on profit of the demands stemming from the two competing scenarios. Obviously, if λ is arbitrarily small, firm 2 will find it optimal to set a substantially high price, although this price might collapse the demand served under the highly competitive scenario, i.e. $b - \frac{p_2 - p_1}{\Delta s} < 0$. However, for higher values of λ , firm 2 could find it optimal to set a lower price in order to keep

competing for $[z, b]$ when $c_1 = c_3$. Therefore, given firm 1's behaviour under $t_1 = a$, if $t_2 = z$, then firm 2 faces a demand:

$$D_2 = \lambda \text{Max} \left\{ 0, b - \frac{p_2 - p_1}{\Delta s} \right\} + (1 - \lambda) \text{Min} \left\{ b - z, b - \frac{p_2 - p_1^M}{\Delta s} \right\}.$$

Finally, given that with positive probability firm 1 has cost $c_1 = c_4$ and sets the monopoly price, if $(1 - \lambda)$ is high enough, firm 2 could find it optimal to set $t_2 = a$, which implies incurring a higher advertising cost, in order to compete¹⁰ with firm 1 by "invading" the segment of the market $[a, z]$, thus yielding a demand: $b - \frac{p_2 - p_1^M}{\Delta s}$, with $\frac{p_2 - p_1^M}{\Delta s} < z$. Once again, if the corresponding p_2 is high enough, this strategy could collapse the demand served when $c_1 = c_3$. Therefore, if $t_2 = a$, firm 2 faces a demand:

$$D'_2 = \lambda \text{Max} \left\{ 0, b - \frac{p_2 - p_1}{\Delta s} \right\} + (1 - \lambda) \left(b - \frac{p_2 - p_1^M}{\Delta s} \right).$$

Having discussed the strategy space of both firms, the following Proposition describes a Bayesian Nash equilibrium of the game:

Proposition 2 *There exists a non-empty set of parameters $[\lambda, a, v, z, s_1, s_2, c_2, c_3, c_4, A_0, A_1]$ for which the following pure strategy Bayesian Nash equilibrium of the price-targeted advertising game exists:*

$$(i) \ t_1 = a, \ p_1^t(c_1) = \begin{cases} \frac{\Delta s[1+a(1-2\lambda)-z(1-\lambda)]+2c_3s_1\lambda+c_2s_2\lambda}{3\lambda} < p_1^M & \text{if } c_1 = c_3 \\ v + as_1 = p_1^M & \text{if } c_1 = c_4 \end{cases}$$

$$(ii) \ t_2 = z, \ p_2^t = \frac{\Delta s[2+a(2-\lambda)-2z(1-\lambda)]+c_3s_1\lambda+2c_2s_2\lambda}{3\lambda}, \text{ and}$$

$$D_2^t = \lambda \left(b - \frac{p_2^t - p_1^t(c_3)}{\Delta s} \right) + (1 - \lambda) (b - z).$$

The intuition behind this Proposition is as follows. Given $t_2 = z$, firm 1's best response, i.e. to monopolize $[a, z]$ or to compete for $[z, b]$, will depend on two aspects. On the one hand, on the price set by its rival, in the sense that a higher (lower) p_2^t increases the incentive to compete (to monopolize). On the other, it is clear that firm 1 has a higher incentive to compete (to monopolize) if production cost is low (high). The key point is that, according to Proposition 2, in equilibrium the price charged by firm 2 can in fact be sufficiently low so that, for a given λ , when $c_1 = c_3$ firm 1 indeed finds it optimal to compete for a higher market share, whereas when $c_1 = c_4$ the firm prefers to concentrate on a smaller segment of the market which can be fully monopolized. Thus, Proposition 2 states that with positive probability, $(1 - \lambda)$, firm 1 will end-up monopolizing

¹⁰In the Appendix we show that when $p_1 = p_1^M$, if $t_2 = a$, firm 2 will always compete with firm 1 for those consumers in $[a, z]$.

the segment of the market $[a, z]$. This result is worthy of note, since it proves that, unlike the results provided in recent works by Galeotti and Moraga (2003) and Iyer *et. al.* (2003), targeted advertising can lead to a *permanent* fragmentation of the market, thus creating a local monopoly.

From the above discussion it follows that a pure strategy Bayesian Nash equilibrium will exist only if p_2^t is not very high. Taking into account that $\frac{\partial p_2^t}{\partial \lambda} < 0$, we therefore can expect that the game will have an equilibrium only for relatively high values of λ . In order to confirm this intuition, we now take a closer look at the existence conditions which are derived in the Appendix. In this regard, a particularly interesting issue is to determine the extent to which existence is compatible with high values of z . Accordingly, Figure 2 illustrates the set of values (λ, z) for which the Bayesian game has a solution, given the value of the remaining parameters corresponding to the case-base market scenario, with $c_3 = 0$, $c_4 = 1$ and $A_1 = A_0(1 + a - z)$. We observe that the parameter space (λ, z) for which the equilibrium exists is indeed compatible only with relatively high levels of λ , $\lambda \geq 0.53$, whereas the value of z can be relatively high, $z = 0.5$. Furthermore, we find that the level of targeting precision can now be very high, given that with probability $(1 - \lambda)$ firm 2 concentrates the advertising approximately on its potential demand,¹¹ $D_2 = b - \frac{p_2^t - p_1^M}{\Delta s} \approx b - z$. Therefore, the model has a solution for high levels of targeting precision and reasonable market conditions.

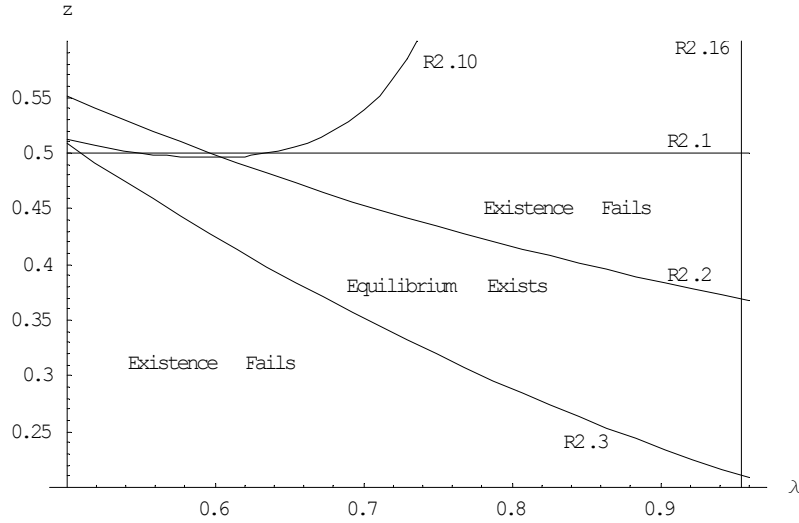


Figure 2: Equilibrium under asymmetric information

Once we have analyzed the existence conditions of the game, our next step is to study how the transition from mass to targeted advertising can affect market outcomes. Let $(p_1^m(c_1), p_2^m)$ denote the optimal pricing strategies when both firms use mass advertising, and let $(\theta_4^m - a)$ be firm 1's

¹¹For example, if $\lambda = 0.6$ and $z = 0.43$, then $b - \frac{p_2^t - p_1^M}{\Delta s} = 0.58$ is very close to $b - z = 0.57$.

equilibrium market share when this firm has high production costs, i.e. $\theta_4^m = \frac{p_2^m - p_1^m(c_4)}{\Delta_s}$. Then:

Proposition 3 (i) If either $z \leq \theta_4^m$, or $s_2 \geq \frac{(c_4 - c_3)\lambda + 2(b - z)}{2(b - z)}s_1$, then $p_1^t(c_j) > p_1^m(c_j)$, $p_2^t > p_2^m$;
(ii) Targeting can lead to a welfare loss.

In order to understand the intuition behind this Proposition, we recall that, under incomplete information, firms can compete under two different scenarios. First, with probability $(1 - \lambda)$ firm 1 has a high production cost and, according to Proposition 2, it charges the monopoly price, i.e. $p_1^t(c_4) = v + as_1$, which, given that under mass advertising the market is assumed to be covered, is higher than $p_1^m(c_4)$. Further, in this case firm 2 faces a demand $[b - z]$, which does not depend on p_2 . Secondly, with probability λ firm 1 has a low production cost and competes with firm 2 for the marginal consumer located within $[b - z]$. The key point is that in the latter scenario, firm 1 perceives that the pattern of price competition with firm 2 is identical to the case in which both firms use mass advertising. That is to say, if $p_1^m(p_2, c_3)$ and $p_1^t(p_2, c_3)$ denote firm 1's reaction functions under mass and targeted advertising, respectively, then we have that $p_1^m(p_2, c_3) \equiv p_1^t(p_2, c_3)$. Firm 2 maximizes profits by taking account of the two possible competing scenarios. With probability λ , we have that $c_1 = c_3$, and under this scenario, competition with mass or targeted advertising is identical, which implies that the optimal price under both advertising strategies would be the same. However, with probability $(1 - \lambda)$, and as compared to mass advertising, if $z \leq \theta_4^m$ firm 2 faces a non-lower and, at the same time, more inelastic demand, which gives this firm an incentive to raise the price. Given that the price charged by firm 2 will be a linear combination of the optimal prices corresponding to the two competitive scenarios, we have that $p_2^t > p_2^m$. Finally, taking into consideration that, when $c_1 = c_3$, it holds that $p_1^m(p_2, c_1) \equiv p_1^t(p_2, c_1)$, and that in our model the high-quality good and the low-quality good are strategic complements, i.e. $\frac{\partial p_1}{\partial p_2} > 0$, we have that $p_1^t(c_3) > p_1^m(c_3)$.

If $z > \theta_4^m$, then, as compared to mass advertising, when $c_1 = c_4$ firm 2 faces a lower and, at the same time, more inelastic demand, in such a way that, a priori, it is not clear whether there is an incentive to raise the price. In this case, Proposition 3 indicates that targeting leads to higher prices if the degree of product heterogeneity (or quality differential) in the market is sufficiently high. We note that this *sufficient* result holds for quite reasonable market conditions: for example, if $a = 0$, $z = \frac{1}{2}$, $c_3 = \frac{1}{3}$ and $c_4 = \frac{2}{3}$, then a sufficient condition is $s_2 \geq \frac{4}{3}s_1$. Nevertheless, in order to shed further light on this issue, Figure 3 represents the existence conditions of the Bayesian game in the plane (s_1, s_2) , together with the equation $p_2^t = p_2^m$, for the base-case market scenario used in Figure 2, with $\lambda = 0.7$ and $z = 0.4$. We can observe that the equilibrium exists *only* for the set of

values (s_1, s_2) such that $p_2^t > p_2^m$, which confirms the intuition provided in Proposition 3 part (i).¹²

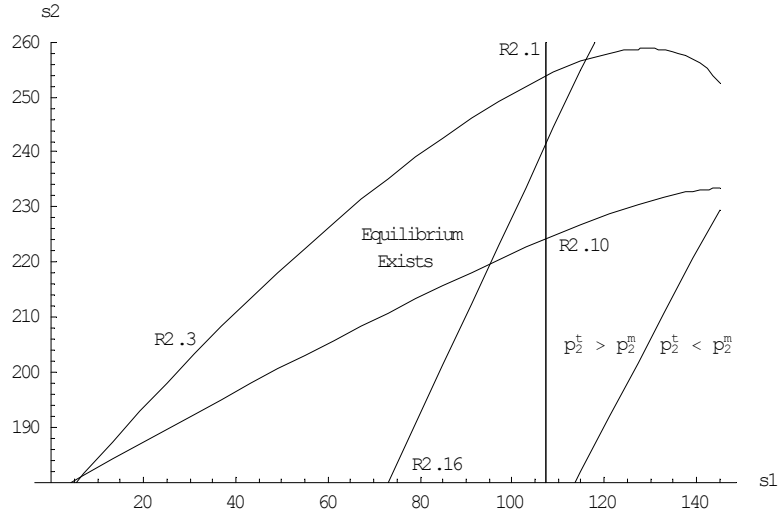


Figure 3: Equilibrium existence and market prices.

Regarding the comparative static results of our model,¹³ it is particularly interesting to note that the extent to which targeted advertising allows firms to raise their prices is inversely related to the level of precision of the targeting technology, i.e. $\frac{\partial p_1^t(c_3)}{\partial z} < 0$, $\frac{\partial p_2^t}{\partial z} < 0$. This result stems from the fact that when $c_1 = c_4$, as targeting becomes more precise, firm 2 faces a lower demand with the same degree of elasticity, which leads to lower market prices. As a result, we also find that when $c_1 = c_4$, it holds that $\frac{\partial \Pi_1^t}{\partial z} > 0$, whereas $c_1 = c_3$ implies $\frac{\partial \Pi_1^t}{\partial z} < 0$. With respect to firm 2, a higher degree of targeting also increases advertising cost efficiency, and so the relationship between firm 2's profits and z is ambiguous.

According to these results, we conclude that targeting not only increases advertising efficiency, but also allows firms to increase their degree of market power. This latter effect has a negative effect on consumer surplus, and so the natural question to ask is whether the transition from mass to targeted advertising might have an adverse effect on social welfare. Obviously, if the market remains covered, targeting results only in higher advertising cost efficiency, and is thus welfare improving. However, if $c_1 = c_4$ and v is sufficiently low (see Restriction 3.1. in the Appendix), then firm 1 charges $p_1^M > v + as_1$, in such a way that the fragment of the market $\left[a, \frac{p_1^M - v}{s_1}\right]$ will not be served. In this case, the final impact of targeting on welfare depends on the comparison between the social gain induced by the higher advertising cost efficiency, and the social loss due

¹²We have conducted extensive simulations of our model, confirming that the existence conditions are incompatible with $p_2^t \leq p_2^m$.

¹³These results are straightforward derivations from the results provided in Proposition 2.

to the typical quantity distortion introduced by the monopolist. We have computed the model to shed light on the relative strength of these two forces, and find that targeting can indeed lead to a welfare loss. However, the critical question is to what extent, or, equivalently, under which market conditions, can we expect this result to hold.

In order to address this issue, in Figure 4 we have compared the existence restrictions of the Bayesian game with those conditions under which welfare decreases, for a market scenario $[z, \lambda, a, v, c_3, c_4, c_2, A_0, A_1] = [0.4, 0.85, 0.1, 140, 0, 1, 0.7, 5, 3.5]$, such that $p_1^M > v + as_1$.

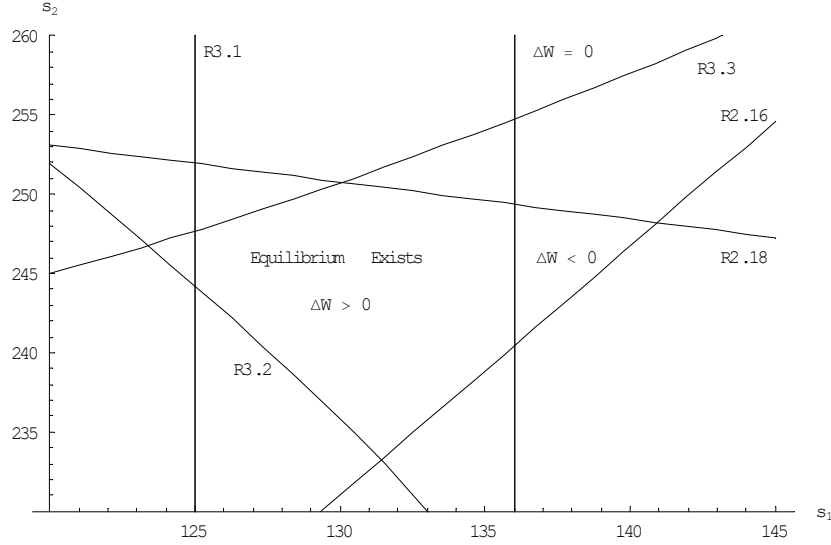


Figure 4: Equilibrium and Welfare

We can observe that the game has a pure strategy Bayesian equilibrium only under very restrictive market conditions, and that the level of welfare decreases only in a reduced proportion of these equilibria. Therefore, we conclude that the probability that targeting leads to a welfare loss is extremely reduced, or equivalently, that targeting has a high potential to be welfare improving.¹⁴

To summarize, our incomplete information model of targeted advertising shows that with probability $(1 - \lambda)$, targeted advertising can lead to a permanent fragmentation of the market. Further, a particularly noteworthy result is that, even if the market is not fragmented, which occurs with probability λ , this advertising technology also leads to higher market prices. However, we conclude that the higher advertising cost efficiency induced by targeting is very likely to compensate for the higher degree of monopoly power, in such a way that this advertising technology offers significant potential for improving welfare.

Finally, we note that, in our model, the probability of market fragmentation is exogenous,

¹⁴Extensive simulations of our model have confirmed this intuition.

and so we cannot examine how this probability relates to market conditions. Furthermore, we have analyzed the impact of targeting on the market outcome only under uniform pricing, which is the mainstream case of most product markets where the good is sold to consumers through traditional retail channels. However, the advances in information technologies, such as the Internet, are increasing firms' ability to price discriminate and target different prices to different groups of consumers. The next section extends the analysis of targeting to address these two questions.

4 Extensions

In this section, we first extend the incomplete information model of targeted advertising developed in section 3, by considering a continuous distribution of firm 1's production cost. Secondly, we analyze competition with both targeted advertising and targeted prices in the complete information framework developed in section 2.

4.1 Equilibrium with a continuous distribution of production costs

We now assume that c_1 is distributed uniformly on $[0, 1]$ and, for the sake of simplicity, that $[a, b] = [0, 1]$. The following Proposition describes the equilibrium under these conditions:

Proposition 4 *a) There exists a non-empty set of parameters $[v, z, s_1, s_2, c_2, A_0, A_1]$ for which the following pure strategy Bayesian Nash equilibrium of the price-targeted advertising game exists:*

$$(i) \ t_1 = 0, \quad p_1^t(c_1) = \begin{cases} \frac{p_2^t + c_1 s_1}{2} & \text{if } c_1 \in [0, c_1^*] \\ v & \text{if } c_1 \in [c_1^*, 1] \end{cases}$$

$$(ii) \ t_2 = z, \quad p_2^t = \frac{4s_2[1 + c_2 c_1^* - (1 - c_1^*) z] + s_1[(c_1^*)^2 - 4 + 4z(1 - c_1^*)]}{6c_1^*},$$

and $D_2^t = c_1^* \left(b - \frac{p_2^t - p_1^t(c_3)}{\Delta s} \right) + (1 - c_1^*)(b - z)$, where c_1^* is implicitly given by

$$(v - c_1^* s_1) z - \frac{\{4s_2[1 + c_2 c_1^* - (1 - c_1^*) z] + s_1[4z(1 - c_1^*) - 4 - 5(c_1^*)^2]\}^2}{24 c_1^* \Delta s} = 0.$$

b) In equilibrium the following relations hold: $\frac{\partial c_1^}{\partial z} < 0$, $\frac{\partial c_1^*}{\partial v} < 0$, $\frac{\partial c_1^*}{\partial c_2} > 0$, $\frac{\partial c_1^*}{\partial A_j} = 0$.*

c) If $s_2 > \frac{5-4z}{4-4z} s_1$, then $p_1^t(c_1) > p_1^m(c_1)$, $p_2^t > p_2^m$.

The distinctive feature of this equilibrium is that, for a given strategy of firm 2, firm 1's best response takes the form of a *cutoff rule* based on a profit indifference condition: it optimally monopolizes the captive market $[a, z]$ for all c_1 above a critical value, c_1^* , and does compete for

the segment of the market $[z, 1]$ for all $c_1 < c_1^*$. Therefore, c_1^* represents the level of c_1 for which the profit achieved by firm 1 with the monopolizing strategy, minus the profit achieved with the competing strategy, equals zero. Table 1 offers several examples of equilibria under different market conditions (note that each column represents a different equilibrium).

Insert Table 1 approximately here.

The interesting feature of this model is that the probability that targeted advertising results in market fragmentation depends on market conditions. In particular, we demonstrate that the probability of monopolization, $(1 - c_1^*)$, is positively related to both z and v , negatively related to c_2 , and does not depend on the advertising costs. The intuition of these comparative statics results is as follows. As the level of targeting precision increases, so the inelastic part of firm 2's demand is smaller which, for a given pricing strategy on the part of firm 1, will lead firm 2 to charge a lower price. At the same time, the captive market that firm 1 can monopolize is greater. As a result of these two effects, and for a given value of the remaining parameters, as z increases, firm 1 finds the monopolization strategy more attractive, relative to the competition strategy. At this point, it is important to note that the equilibrium exists only if, in a neighborhood of c_1^* , the *cut off* equation is increasing in c_1 (see Appendix). Therefore, as a result of a higher z , the value of c_1^* must decrease in order to restore the profit indifferent condition, and so we conclude that a higher level of targeting precision increases the probability of market fragmentation, $(1 - c_1^*)$. Following the same intuition, we also conclude that both a higher value of v and a lower value of c_2 make monopolization a more attractive strategy, thus leading to a higher probability of market fragmentation. Finally, the *cutoff* rule, and therefore the value of c_1^* , does not depend on A_0 and A_1 and so, for a given equilibrium, these two variables do not seem to affect the probability of monopolization. However, it should be noted that the costs of the advertising campaigns are crucial for the existence of an equilibrium and, therefore, the values of A_0 and A_1 will indirectly affect the possibility of market fragmentation.

Finally, Proposition 4 provides a sufficient condition under which targeting leads to higher prices. Notice that this condition is, in fact, quite weak (for example, if $z = 0.5$, it is sufficient that $s_2 > 1.5 s_1$, if $z = 0.6$, the condition is $s_2 > 1.625 s_1$ and so on) which suggests that targeting will indeed allow firms to increase their degree of market power. We have confirmed this intuition with extensive simulations of our model, which indicate that the existence conditions are incompatible with $p_i^t \leq p_i^m$, $i = 1, 2$.

4.2 Targeted Advertising and Price Discrimination

This section analyzes how targeted advertising can be used as a price discrimination device. In particular, we show that there exists a *pure strategy* Nash equilibrium of the pricing-targeted advertising game in which firm 2 advertises a price p_2^t to the segment of the market $[z, b]$, whilst firm 1 uses both the mass media, $t_1 = a$, where it announces a high price, p_1^t , and the specialized media, $\hat{t}_1 = z$, where it inserts discount coupons which allow high valuation consumers to purchase the low-quality good at a reduced price, $\hat{p}_1^t$. For the sake of simplicity, in this section we assume that $C_i(s_i) = cs_i$, i.e. $c_1 = c_2 = c$, and that the targeting technology allows firm 2 to focus the campaign exactly on its potential demand under mass advertising, i.e. $z = \theta^m$.

Proposition 5 *There exists a non-empty set of parameters $[v, a, s_1, s_2, c, A_0, A_1]$ for which the following pure strategy Nash equilibrium of the price-advertising game exists:*

$$(i) \left[t_2 = \theta^m; p_2^t = \frac{(5b-a)\Delta s + 4cs_1 + 5cs_2}{9} \right], \text{ with } p_2^t < p_2^m.$$

$$(ii) \left[(t_1 = a, p_1^t = v + as_1); \left(\hat{t}_1 = \theta^m, \hat{p}_1^t = \frac{(b-2a)\Delta s + 8cs_1 + cs_2}{9} \right) \right], \text{ with } p_1^t = p_1^M, \text{ and } \hat{p}_1^t < p_1^m.$$

In this equilibrium it holds that $\Pi_1^t > \Pi_1^m$ and $\Pi_2^t \geq \Pi_2^m$.

In order to better understand this Proposition, let us take the mass advertising equilibrium as the starting point of our discussion. If $z = \theta^m$, the proof of Lemma 1 shows that, given $(t_1 = a, p_1^m)$, firm 2 will set $t_2 = z = \theta^m$, thus leaving the segment of the market $[a, z]$ uninformed. The key point is that, if price discrimination is feasible, then, given $t_2 = \theta^m$, firm 1 can segment the market by distinguishing two types of demand for the low-quality good: (i) a high demand, which stems from those consumers who have imperfect information, and that can be reached by advertising the product in the mass media, and (ii) a low demand, which stems from perfectly informed consumers, and that can be reached by advertising the good in the specialized media. Proposition 5 states that firm 1 can find it optimal to monopolize the captive market and, simultaneously, to “invade” the rival’s market by inserting discount coupons in the specialized advertising media. The distinctive feature of this advertising strategy is that firm 1 will be able to capture some fully informed consumers with a relatively high taste for quality and, therefore, with a high valuation of the high-quality good, given that these consumers are attracted by the substantially lower price at which they can now purchase the low-quality good. Taking into account this low price, $\hat{p}_1^t$, it is obvious that firm 1 will find price discrimination optimal only if the cost of the specialized media is sufficiently low. In this setting, firm 2 has two possible responses: First, it can advertise a low price $p_2 < p_2^t$ in the mass media, $t_2 = a$, thus achieving a larger market share and, secondly, it can simply accommodate the more intense competition induced by the use of coupons by advertising

p_2^t in $t_2 = \theta^m$. Given the high value of p_1^t , the latter strategy will be optimal only if the shift from mass to targeted advertising generates substantial savings in advertising costs. Accordingly, we can expect that the equilibrium described in Proposition 5 will exist only if the cost of the mass media is sufficiently high and the cost of the specialized media sufficiently low. Tables 2 and 3 provide some examples of two type of equilibria with price discrimination, related to the nature of firm 2's demand (see the Appendix), which confirm this intuition.

Insert Tables 2 and 3 approximately here.

We must note that the equilibria in Table 2 are compatible with $\frac{\Delta A}{A_0} \leq (z - a)$, whereas those in Table 3 are compatible only with $\frac{\Delta A}{A_0} > (z - a)$. Regarding the effect of targeting on the market outcome, notice that, under price discrimination, the transition from mass to targeted advertising always benefits firm 1, given that this firm can monopolizes the captive market and achieve an additional profit from the use of coupons. By contrast, the impact on firm 2's profit could be positive or negative, depending on whether the gain in profits due to the savings in advertising costs exceeds the loss of profits due to the more intense competition induced by couponing. In this respect, our simulation suggest that, for reasonable market conditions, targeting is very likely to reduce the profits achieved by the high-quality firm. With regard to consumers, it is clear that those who have imperfect information are worse-off with targeted advertising, given that they end-up paying the monopoly price. However, an interesting point is that *all* consumers who are fully informed pay a lower price and, therefore, are better-off, due to either the use of coupons or the more intense price competition in which both firms engage.

Finally, we can use this model of price discrimination to briefly discuss the optimal supply of quality under targeted advertising. Let us assume that, before competition takes place, firms can develop quality $s_i \in [\underline{s}, \bar{s}]$ at zero cost. We note that $\frac{\partial \Pi_1^t}{\partial s_2} > 0$ (see Appendix), and so $s_2 = \bar{s}$, whereas the sign of $\frac{\partial \Pi_1^t}{\partial s_1} = (a - c)\left(\frac{b-2a+c}{3}\right) - \frac{2[b-2a+c]^2}{81}$ is, a priori, ambiguous. However, the simulations of the model (see Tables 2 and 3), conducted over a wide range of parameter values, yield that the conditions under which the Nash equilibrium exists are incompatible with $a > c$, suggesting that, in practice, $\frac{\partial \Pi_1^t}{\partial s_1} < 0$. This result confirms, under quite extreme conditions, the so-called "*Principle of Maximum Differentiation*," given that, although firm 1 monopolizes the captive market and the optimal price is positively correlated to s_1 , $\frac{\partial p_1^M}{\partial s_1} > 0$, the optimal supply of quality is nevertheless minimum, i.e. $s_1 = \underline{s}$.

5 Conclusions

In this paper we have analyzed the strategic use of targeted advertising. In line with some of the recent literature on informative advertising, we analyze a particular targeting technology which allows firms to reach only the most eager consumers. Under this definition of targeting, the natural way to model price competition is in a vertically differentiated market, and so we examine targeted advertising when firms offer products with different levels of quality. In particular, we propose a strategic game of pricing and targeted advertising in which the cost of the low-quality firm is private information. In this setting, we demonstrate that, for relatively high levels of targeting precision, there exists a pure strategy Bayesian Nash equilibrium which is consistent with *permanent* market fragmentation. A particularly noteworthy result is that, regardless of whether or not the market is fragmented, and as compared to mass advertising, targeted advertising always leads to higher market prices, and so this advertising strategy allow both firms to exercise a higher degree of market power. Further, we show that market fragmentation can induce a welfare loss, due to the typical quantity distortion introduced by a monopolist, and that targeted advertising might have a detrimental effect on welfare, although this possibility is of very limited practical relevance.

In a first extension of our model, we endogeneize the probability that targeting results in market fragmentation, in order to study how this probability relates to market conditions. In this regard, we show that the probability of monopolization increases if: (i) the cost of the high-quality firm decreases, (ii) the degree of targeting precision increases and, finally, (iii) if consumers' valuation of the good increases.

In a second extension of our work, we analyze the relationship between targeted advertising and targeted prices in a complete information framework. In this setting, we show that, if both the level of targeting precision is high and the cost of the specialized media is low, then there exists a pure strategy Nash equilibrium of the price and targeted advertising game in which the low-quality firm monopolizes a segment of the market and, simultaneously, captures a segment of its rival's "natural" market by price discriminating with discount coupons. Finally, building on this equilibrium, we analyze the optimal supply of quality, finding that price competition with vertically differentiation products and targeted advertising leads to maximum product differentiation.

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6 Appendix

Proof of Lemma 1:

If $t_1 = t_2 = a$ and firms compete for the fully informed marginal consumer $\theta = \frac{p_2 - p_1}{\Delta s}$, profits are given by¹⁵

$$\Pi_1 = (p_1 - c_1 s_1) \left(\frac{p_2 - p_1}{\Delta s} - a \right) - A_0, \quad (1)$$

$$\Pi_2 = (p_2 - c_2 s_2) \left(b - \frac{p_2 - p_1}{\Delta s} \right) - A_0. \quad (2)$$

¹⁵For details on this equilibrium, see Tirole (1988).

Differentiating these expressions with respect to p_1 and p_2 we obtain the reactions functions:

$$p_1^m(p_2) = \frac{p_2 - a\Delta s + c_1 s_1}{2}, \quad (3)$$

$$p_2^m(p_1) = \frac{p_1 + b\Delta s + c_2 s_2}{2}. \quad (4)$$

The intersection of (3) and (4) yields the unique Nash equilibrium of the pricing game: $p_1^m = \frac{\Delta s(1-a)+2c_1 s_1+c_2 s_2}{3}$, $p_2^m = \frac{\Delta s(2+a)+2c_2 s_2+c_1 s_1}{3}$, with profits $\Pi_1^m = \frac{[\Delta s(1-a)+c_2 s_2-c_1 s_1]^2}{9\Delta s} - A_0$, and $\Pi_2^m = \frac{[\Delta s(2+a)+c_1 s_1-c_2 s_2]^2}{9\Delta s} - A_0$. Next, let us assume that firms can target their ads on $z \leq \theta^m = \frac{p_2^m - p_1^m}{\Delta s}$. Then, given $(t_1 = a, p_1^m)$, firm 2's profits under targeted advertising are: $\Pi_2(p_2, z) = (p_2 - c_2 s_2) \text{Min} \left\{ b - \frac{p_2 - p_1^m}{\Delta s}, b - z \right\} - A_1(z)$. Assume, for the moment, that $\text{Min} \left\{ b - \frac{p_2 - p_1^m}{\Delta s}, b - z \right\} = b - \frac{p_2 - p_1^m}{\Delta s}$. In this case, the price that maximizes $\Pi_2(p_2, z) = (p_2 - c_2 s_2) (b - \frac{p_2 - p_1^m}{\Delta s}) - A_1(z)$ is $p_2 = p_2^m$, and so it holds that $b - \frac{p_2^m - p_1^m}{\Delta s} \leq b - z$. Finally, $\frac{\partial \Pi_2(p_2^m, z)}{\partial z} = -A_1'(z) > 0$ which implies that, given firm 1' pricing-advertising strategy, firm 2's best response is $t_2 = z$. Therefore, $t_1 = t_2 = a$ cannot be part of an equilibrium. Q.E.D.

Proof of Proposition 1:

In order to prove that E1 and E2 are an equilibrium, we need to check that, given their rival's strategy profile, both firms do not have an incentive to deviate and, further, that sellers achieve positive profits.

Let us start with E1= $[(t_1 = a, p_1^t = p_1^m); (t_2 = z, p_2^t = p_2^m)]$:

- Given $(t_1 = a, p_1^t = p_1^m)$, if $z < \theta^m$, the proof of Lemma 1 shows that firm 2's best response is indeed $(t_2 = z, p_2^t = p_2^m)$, and so this firm does not have any profitable deviation.

- Given $(t_2 = z, p_2^t = p_2^m)$, if $t_1 = a$ and firm 1 competes for the segment of the market $[z, b]$, then the demand is $D_1 = \left(\frac{p_2^m - p_1}{\Delta s} - a \right)$ and profits are identical to (1), which implies $p_1^t = p_1^m$. Firm 1 has two possible deviations from this strategy profile: (i) One, it can deviate by advertising the monopoly price, p_1^M , in $t_1 = a$. Straightforward calculations yield $D_1^M = \text{Min} \left\{ z - \frac{p_1 - v}{s_1}, z - a \right\}$ and the monopoly price:

$$p_1^M = \begin{cases} \frac{v + c_1 s_1 + z s_1}{2} & \text{if } v < s_1(z + c_1 - 2a) \\ v + a s_1 & \text{if } v \geq s_1(z + c_1 - 2a) \end{cases}$$

and profits are, $\Pi_1^M(c_1) = (p_1^M - c_1 s_1) D_1^M - A_0$. Therefore, firm 1 will not deviate if $\Pi_1^M < \Pi_1^t = \Pi_1^m$, that is to say if:

Restriction 1.1. $\frac{[v + z s_1 - c_1 s_1]^2}{4 s_1} < \frac{[\Delta s(1-a) + c_2 s_2 - c_1 s_1]^2}{9 \Delta s}$, when $v < s_1(z + c_1 - 2a)$

Restriction 1.2. $(v + a s_1 - c_1 s_1)(z - a) < \frac{[\Delta s(1-a) + c_2 s_2 - c_1 s_1]^2}{9 \Delta s}$, when $v \geq s_1(z + c_1 - 2a)$

(ii) Two, firm 1 can target the ads by setting $t_1 = z$. Under this advertising strategy, profits are $\Pi_1 = (p_1 - c_1 s_1) \left(\frac{p_1^m - p_1}{\Delta s} - z \right) - A_1$. Differentiating this expression yields $p_1^s = \frac{\Delta s(2+a-3z)+2c_2 s_2+4c_1 s_1}{6}$, and $\Pi_1^s = \frac{[\Delta s(2+a-3z)+2c_2 s_2-2c_1 s_1]^2}{36\Delta s} - A_1$. This deviation will not be profitable if $\Pi_1^s < \Pi_1^t$. Accordingly, an equilibrium exists if and only if the following set of restrictions hold: **Restriction 1.1.**, **Restriction 1.2.** and:

$$\textbf{Restriction 1.3. } \frac{[\Delta s(2+a-3z)+2c_2 s_2-2c_1 s_1]^2}{36\Delta s} - A_1 < \frac{[\Delta s(1-a)+c_2 s_2-c_1 s_1]^2}{9\Delta s} - A_0$$

$$\textbf{Restriction 1.4. } \Pi_1^m = \frac{[\Delta s(1-a)+c_2 s_2-c_1 s_1]^2}{9\Delta s} - A_0 > 0$$

$$\textbf{Restriction 1.5. } \Pi_2^m = \frac{[\Delta s(2+a)+c_1 s_1-c_2 s_2]^2}{9\Delta s} - A_0 > 0$$

$$\textbf{Restriction 1.6. } U = v + a s_1 - p_1^m > 0$$

Therefore, we must show that the parameter space imposed by these restrictions is not empty. The simplest way to address this task is to reduce the number of parameters in the model by considering a base-case market scenario: $[a, v, s_1, s_2, c_1, c_2, A_0] = [0, 150, 100, 240, 0.5, 0.5, 10]$. Under these parameter values, it is straightforward to show that Restrictions 1.1, 1.4, 1.5 and 1.6 are redundant, in such a way that the above set of restrictions hold if and only if: $z \leq 0.35$ (R. 1.2.), and $z \geq 0.01428$ ($70 - 11.83215\sqrt{25 + A_1}$) (R. 1.3.). Therefore, if, for example, $A_1 = 6$, then for all $0.058 \leq z \leq 0.350$ the equilibrium exists.

Next, we turn to analyze $E2 = [(t_1 = z, p_1^t); (t_2 = z, p_2^t)]:$

Under these targeting strategies, firms' profits are

$$\Pi_1 = (p_1 - c_1 s_1) \left(\frac{p_2 - p_1}{\Delta s} - z \right) - A_1, \quad (5)$$

$$\Pi_2 = (p_2 - c_2 s_2) \left(b - \frac{p_2 - p_1}{\Delta s} \right) - A_1. \quad (6)$$

The unique Nash equilibrium of the pricing game yields, $p_1^t = \frac{[\Delta s(1+a-2z)+2c_1 s_1+c_2 s_2]}{3}$, $p_2^t = \frac{[\Delta s(2+2a-z)+2c_2 s_2+c_1 s_1]}{3}$, $\Pi_1^t = \frac{[\Delta s(1+a-2z)+c_2 s_2-c_1 s_1]^2}{9\Delta s} - A_1$, and $\Pi_2^t = \frac{[\Delta s(2+2a-z)+c_1 s_1-c_2 s_2]^2}{9\Delta s} - A_1$. Note that, in this case, both firms can deviate only by setting $t_i = a$. We now analyze the conditions under which this deviation is not profitable.

- Given $(t_2 = z, p_2^t)$, if $t_1 = a$ firm 1 has two possible deviations: (i) One, it can advertise the monopoly price, p_1^M , which yields a profit $\Pi_1^M(c_1) = (p_1^M - c_1 s_1)D_1^M - A_0$. This deviation is not profitable if $\Pi_1^M < \Pi_1^t$, that is to say, if:

$$\textbf{Restriction 1.7. } \frac{[v+zs_1-c_1 s_1]^2}{4s_1} - A_0 < \frac{[\Delta s(1+a-2z)+c_2 s_2-c_1 s_1]^2}{9\Delta s} - A_1,$$

when $v < s_1(z + c_1 - 2a)$

$$\textbf{Restriction 1.8. } (v + a s_1 - c_1 s_1) (z - a) - A_0 < \frac{[\Delta s(1+a-2z)+c_2 s_2-c_1 s_1]^2}{9\Delta s} - A_1,$$

when $v \geq s_1(z + c_1 - 2a)$

(ii) Two, it can compete for $[z, b]$ by advertising a price p_1^s , which is determined by $Max_{p_1} \Pi_1 = (p_1 - c_1 s_1) \left(\frac{p_2^t - p_1}{\Delta s} - a \right) - A_0$. The solution of this problem yields $p_1^s = \frac{\Delta s(2-a-z)+2c_2 s_2+4c_1 s_1}{6}$, $\Pi_1^s = \frac{[\Delta s(2-a-z)+2c_2 s_2-2c_1 s_1]^2}{36\Delta s} - A_0$, and firm 1 will not deviate if $\Pi_1^s < \Pi_1^t$.

• Given $(t_1 = z, p_1^t)$, if $t_2 = a$, firm 2 faces a demand $D_2 = (z - a) + Max \left[b - \frac{p_2 - p_1^t}{\Delta s}, 0 \right]$. Therefore, we must distinguish two situations: (i) One, if $b < \frac{p_2 - p_1^t}{\Delta s}$, firm 2 will monopolize $[z - a]$ which yields $D_2^M = Min \left\{ z - \frac{p_2 - v}{s_2}, z - a \right\}$,

$$p_2^M = \begin{cases} \frac{v + c_2 s_2 + z s_2}{2} & \text{if } v < s_2(z + c_2 - 2a) \\ v + a s_2 & \text{if } v \geq s_2(z + c_2 - 2a) \end{cases}$$

and profits are, $\Pi_2^M(c_2) = (p_2^M - c_2 s_2) D_2^M - A_0$. Therefore, firm 2 will not deviate if $\Pi_2^M < \Pi_2^t$, that is to say, if:

Restriction 1.9. $\frac{[v + z s_2 - c_2 s_2]^2}{4s_2} - A_0 < \frac{[\Delta s(2+2a-z)+c_1 s_1 - c_2 s_2]^2}{9\Delta s} - A_1$,

when $v < s_2(z + c_1 - 2a)$

Restriction 1.10. $(v + a s_2 - c_2 s_2)(z - a) - A_0 < \frac{[\Delta s(2+2a-z)+c_1 s_1 - c_2 s_2]^2}{9\Delta s} - A_1$,

when $v \geq s_2(z + c_1 - 2a)$

(ii) Two, if $\frac{p_2 - p_1^t}{\Delta s} < b$, then firm 2 will compete for $[z, b]$ by charging a price p_2^s , which is the solution of $Max_{p_2} \Pi_2 = (p_2 - c_2 s_2) \left[(z - a) + \left(b - \frac{p_2 - p_1^t}{\Delta s} \right) \right] - A_0$. Differentiating this expression yields $p_2^s = \frac{\Delta s(4+a+z)+2c_1 s_1+4c_2 s_2}{6}$, and $\Pi_2^s = \frac{[\Delta s(4+a+z)+2c_1 s_1-2c_2 s_2]^2}{36\Delta s} - A_0$, and firm 2 will not deviate if $\Pi_2^s < \Pi_2^t$.

Therefore, an equilibrium exists if and only if the following set of restrictions hold: **1.4., 1.5., 1.6., 1.7., 1.8., 1.9., 1.10** and

Restriction 1.11. $\frac{[\Delta s(2-a-z)+2c_2 s_2-2c_1 s_1]^2}{36\Delta s} - A_0 < \frac{[\Delta s(1+a-2z)+c_2 s_2-c_1 s_1]^2}{9\Delta s} - A_1$

Restriction 1.12. $\frac{[\Delta s(4+a+z)+2c_1 s_1-2c_2 s_2]^2}{36\Delta s} - A_0 < \frac{[\Delta s(2+2a-z)+c_1 s_1-c_2 s_2]^2}{9\Delta s} - A_1$

Restriction 1.13. $\Pi_1^t = \frac{[\Delta s(1+a-2z)+c_2 s_2-c_1 s_1]^2}{9\Delta s} - A_1 > 0$

Restriction 1.14. $\Pi_2^t = \frac{[\Delta s(2+2a-z)+c_1 s_1-c_2 s_2]^2}{9\Delta s} - A_1 > 0$.

For the base-case market scenario, all these restrictions hold if $z \leq 0.04285 (70 - 6.8313\sqrt{95 + A_1})$ (R.1.12.). Therefore, if, for example, $A_1 = 6$, then for all $z \leq 0.057$ the equilibrium exists. Finally, we can note that $p_2^t < p_2^m$, whereas, given $(1 - a) > (1 + a - 2z)$, we have that $p_1^t < p_1^m$. This completes the proof.

Proof of Lemma 2:

For the targeting strategy $(t_1 = a, t_2 = z)$, if firm 1 competes for fully informed consumers in $[z, b]$, then we know that the unique equilibrium prices are (p_1^m, p_2^m) , with $\theta^m = \frac{p_2^m - p_1^m}{\Delta s}$. However, this is not an equilibrium strategy, given that if $t_2 = z = \theta^m$ and $p_2 = p_2^m$, then it is clear that firm 1's best response will be to monopolize the captive market by setting $p_1^M = \text{Max} \left[v + as_1, \frac{v+s_1(z+c_1)}{2} \right] > p_1^m$, in such a way that $\theta^M = \frac{p_2^m - p_1^M}{\Delta s} < \theta^m$ and the demand served by firm 1 is $D_1^M = \text{Min} \left[(z-a), \frac{v+s_1(z-c_1)}{2s_1} \right]$. As a result of this monopolization strategy, if $t_2 = z$, firm 2' demand is $D_2 = \text{Min} \left[b-z, b - \frac{p_2 - p_1^M}{\Delta s} \right]$. Given that from (4) we know that $\frac{\partial p_2}{\partial p_1} < 1$, the solution of $\text{Max}_{p_2} (p_2 - c_2 s_2) (b - \frac{p_2 - p_1^M}{\Delta s}) - A_1$ yields $\frac{p_2 - p_1^M}{\Delta s} < \theta^m$. Therefore, $D_2 = b - z$, and firm 2 will respond to p_1^M by charging the maximum price that the marginal consumer, $z = \theta^m$, is willing to pay i.e. $p_2' = p_1^M + \Delta s \theta^m$, in such a way that $\frac{p_2' - p_1^M}{\Delta s} = \theta^m$. Given $(t_2 = z, p_2')$, firm 1 can either monopolize $[a, z]$, which yields a benefit $\Pi_1^M = (p_1^M - c_1 s_1) D_1^M - A_0$, or compete for $[z, b]$, which implies to maximize $\Pi_1' = (p_1 - c_1 s_1) (\frac{p_2' - p_1}{\Delta s} - a) - A_0$. The solution of this problem yields $p_1' = \frac{p_2' - a\Delta s + c_1 s_1}{2}$ and profits $\Pi_1' = \frac{[p_1^M + (z-a)\Delta s - c_1 s_1]^2}{4\Delta s} - A_0$. Some calculations yield that $\Pi_1^M > \Pi_1'$ implies $(p_1^M - c_1 s_1)^2 + (z-a)^2 \Delta s^2 + (p_1^M - c_1 s_1) \Delta s [2(z-a) - 4D_1^M] < 0$. Taking into consideration that $D_1^M \leq z-a$, we have that $2(z-a) - 4D_1^M \geq -2(z-a)$, and so $0 > (p_1^M - c_1 s_1)^2 + (z-a)^2 \Delta s^2 + (p_1^M - c_1 s_1) \Delta s [2(z-a) - 4D_1^M] \geq (p_1^M - c_1 s_1)^2 + (z-a)^2 \Delta s^2 - 2(p_1^M - c_1 s_1) \Delta s (z-a) = [(p_1^M - c_1 s_1) - \Delta s(z-a)]^2$, which constitutes a contradiction. This shows that firm 1's best response is to compete for $[z, b]$. Finally, if both firms compete for the fully informed consumers in $[z, b]$, the unique Nash equilibrium of the pricing game is (p_1^m, p_2^m) which, as we have already shown, cannot be part of an equilibrium.

Proof of Proposition 2:

We must show that, given their rival's strategy, neither firm has an incentive to deviate from the equilibrium strategy profile.

- We begin the proof by considering that $t_1 = a$. In this case, and given $(t_2 = z, p_2^t)$, if firm 1 monopolizes $[z-a]$ and $v \geq s_1(z + c_1 - 2a)$, then $D_1 = (z-a)$, $p_1^M = v + as_1$ and $\Pi_1^M = (v + as_1 - c_1 s_1) (z-a) - A_0$. By contrast, if firm 1 competes for $[z, b]$, then its reaction function is $p_1^t(p_2, c_1) = \frac{p_2 - a\Delta s + c_1 s_1}{2}$, and profits are $\Pi_1^t = \frac{[p_2^t - a\Delta s - c_1 s_1]^2}{4\Delta s} - A_0$. Therefore, if $t_1 = a$, the pricing strategy $p_1^t(c_1)$ described in Proposition 2 can be optimal only if:

Restriction 2.1. $v \geq s_1(z + c_4 - 2a)$

Restriction 2.2. $\frac{[p_2^t - a\Delta s - c_3 s_1]^2}{4\Delta s} > (v + as_1 - c_3 s_1)(z-a)$

Restriction 2.3. $\frac{[p_2^t - a\Delta s - c_4 s_1]^2}{4\Delta s} < (v + as_1 - c_4 s_1)(z-a)$

Next, we check that $t_1 = a$ is indeed optimal. Given firm 2's pricing-advertising strategy, if

$t_1 = z$, firm 1 will always compete for all the potential customers in $[z, b]$. Under these conditions, firm 1 will not deviate from $t_1 = a$ if

$$\textbf{Restriction 2.4. } \frac{[p_2^t - z\Delta s - c_3 s_1]^2}{4\Delta s} - A_1 < \frac{[p_2^t - a\Delta s - c_3 s_1]^2}{4\Delta s} - A_0$$

$$\textbf{Restriction 2.5. } \frac{[p_2^t - z\Delta s - c_4 s_1]^2}{4\Delta s} - A_1 < (v + a s_1 - c_4 s_1)(z - a) - A_0$$

• We now turn to analyze firm 2's optimal strategy. Given $(t_1 = a, p_1^t(c_1))$, if $t_2 = z$, under the following conditions:

$$\textbf{Restriction 2.6. } b - \frac{p_2^t - p_1^t(c_3)}{\Delta s} > 0$$

$$\textbf{Restriction 2.7. } b - z < b - \frac{p_2^t - p_1^M}{\Delta s},$$

firm 2 faces the problem:

$$\text{Max}_{p_2} E\Pi_2 = (p_2 - c_2 s_2) \left[\lambda \left(b - \frac{p_2 - p_1^t(c_3)}{\Delta s} \right) + (1 - \lambda)(b - z) \right] - A_1,$$

which yields $p_2^t = \frac{\Delta s[2+a(2-\lambda)-2z(1-\lambda)]+c_3 s_1 \lambda+2c_2 s_2 \lambda}{3\lambda}$, and so $p_1^t(c_3) = \frac{\Delta s[1+a(1-2\lambda)-z(1-\lambda)]+2c_3 s_1 \lambda+c_2 s_2 \lambda}{3\lambda}$.

Further, expected profits are: $E\Pi_2^t = \frac{[\Delta s(2(1+a-z)-\lambda(a-2z))-\lambda c_2 s_2 + \lambda c_3 s_1]^2}{9\lambda \Delta s} - A_1$.

Firm 2 has three possible deviations from $(t_2 = z, p_2^t)$:

(i) One, it can set $(t_2 = z, p_2)$ such that $\text{Max} \{0, b - \frac{p_2 - p_1}{\Delta s}\} = 0$. In this case, firm 2's problem is: $\text{Max}_{p_2} \Pi_2 = (p_2 - c_2 s_2) \text{Min} \left\{ b - z, b - \frac{p_2 - p_1^M}{\Delta s} \right\} (1 - \lambda) - A_1$. The solution of this problem yields:

$$p_2^s = \begin{cases} \frac{p_1^M + b\Delta s + c_2 s_2}{2} & \text{if } v < (b - 2z)\Delta s + c_2 s_2 - a s_1 \\ p_1^M + z\Delta s & \text{if } v \geq (b - 2z)\Delta s + c_2 s_2 - a s_1 \end{cases}$$

and so firm 2 will not deviate if:

$$\textbf{Restriction 2.8. } \frac{[p_1^M + b\Delta s - c_2 s_2]^2}{4\Delta s} (1 - \lambda) - A_1 < E\Pi_2^t, \text{ when } v < (b - 2z)\Delta s + c_2 s_2 - a s_1$$

$$\textbf{Restriction 2.9. } (p_1^M + z\Delta s - c_2 s_2)(b - z)(1 - \lambda) - A_1 < E\Pi_2^t, \\ \text{when } v \geq (b - 2z)\Delta s + c_2 s_2 - a s_1$$

(ii) Two, if $t_2 = a$, and $D_2' = \lambda \left(b - \frac{p_2 - p_1}{\Delta s} \right) + (1 - \lambda) \left(b - \frac{p_2 - p_1^M}{\Delta s} \right)$, profit maximization yields

$$p_2^d = \frac{[3v(1 - \lambda) + s_2(4 + 4a + 3c_2 - z + \lambda(z + c_2 - 2a)) - s_1(4 + a - z + \lambda(z + a - 2c_3))]}{6}$$

$$E\Pi_2^d = \frac{1}{36\Delta s} [3v(-1 + \lambda) + s_2(-4 + 3c_2 + z + 2a(\lambda - 2) - \lambda(c_2 + z)) + \\ + s_1(4 + a - z + \lambda(z + a - 2c_3))]^2 - A_0.$$

Firm 2 will not deviate if $E\Pi_2^d < E\Pi_2^t$.

(iii) Three, if $t_2 = a$, and $D'_2 = (1 - \lambda) \left(b - \frac{p_2 - p_1^M}{\Delta s} \right)$, with $\frac{p_2 - p_1^M}{\Delta s} < z$, then profit maximization yields $p_2^k = \frac{p_1^M + b\Delta s + c_2 s_2}{2}$ and $\Pi_2^k = \frac{[p_1^M + b\Delta s - c_2 s_2]^2}{4\Delta s} (1 - \lambda) - A_0$. Firm 2 will not deviate if $\Pi_2^k < E\Pi_2^t$.

Finally, using the equilibrium strategy and Restrictions **2.8.** and **2.9.**, it is straightforward to check that if $t_2 = a$ and $c_1 = c_4$, those pricing strategies in which firm 2's does not compete for those consumers in $[a, z]$ cannot be a profitable deviation.

In summary, an equilibrium exists if and only if the following set of restrictions hold: **Restriction 2.1., 2.2., 2.3., 2.4., 2.5., 2.6., 2.7., 2.8., 2.9.**, and:

Restriction 2.10. $E\Pi_2^d < E\Pi_2^t$

Restriction 2.11. $\Pi_2^k < E\Pi_2^t$

Restriction 2.12. $\Pi_1^M = (v + as_1 - c_4 s_1) (z - a) - A_0 > 0$

Restriction 2.13. $\Pi_1^t(c_3) = \frac{[p_2^t - c_3 s_1 - a\Delta s]^2}{4\Delta s} - A_0 > 0$

Restriction 2.14. $E\Pi_2^t = \frac{[\Delta s(2(1+a-z) - \lambda(a-2z)) - \lambda c_2 s_2 + \lambda c_3 s_1]^2}{9\lambda\Delta s} - A_1 > 0$

Restriction 2.15. $\Pi_1^m(c_3) = \frac{[2(1-a+c_2)s_2 + s_1(-2+2a+c_4(1-\lambda)+c_3(-3+\lambda))]^2}{36\Delta s} - A_0 > 0$

Restriction 2.16. $\Pi_1^m(c_4) = \frac{[2(1-a+c_2)s_2 + s_1(-2+2a+c_3\lambda-c_4(2+\lambda))]^2}{36\Delta s} - A_0 > 0$

Restriction 2.17. $E\Pi_2^m = \frac{[(-2-a+c_2)s_2 + s_1(2+a-c_4(1-\lambda)-c_3\lambda)]^2}{9\Delta s} - A_0 > 0$

Restriction 2.18. $U = v + as_1 - p_1^m(c_4) > 0$

For the base-case market scenario, if we consider $A_1 = (1 + a - t)A_0$ and $[c_3, c_4] = [0, 1]$, all the restrictions, with the exception of 2.1., 2.2, 2.3, 2.10 and 2.16, are redundant. Therefore, the above set of restrictions hold if $z \leq \frac{[1+3.9642\lambda^2 + \lambda(-0.1428-3.8705\sqrt{0.6436-0.0919\lambda+\lambda^2})]}{(1-\lambda)^2}$ (R.2.2), $z \geq \frac{[1+1.8214\lambda^2 + \lambda(-1.2142-1.8087\sqrt{0.9824-1.1929\lambda+\lambda^2})]}{(1-\lambda)^2}$ (R. 2.3),

$$z \leq \frac{-16-13.8571\lambda^2+2.3571\lambda^3+\lambda[28.7857+3.0653x10^{-8}\sqrt{(\frac{1}{\lambda}(\lambda-6.3331x10^{16})(\lambda-0.8864)(\lambda-0.7841)(4.1383-4.0053\lambda+\lambda^2))}]}{(\lambda-16)(\lambda-1)^2}$$

(R.2.10), $\lambda \leq 0.9550$ (R. 2.16) and, finally, $z \leq 0.5$ (R.2.1). Therefore, if, for example, $\lambda = 0.7$, then for all $0.3518 \leq z \leq 0.4535$ the equilibrium exists. This completes the proof.

Proof of Proposition 3:

(i) If $t_1 = t_2 = a$, under incomplete information firms' profits are:

$$\Pi_1 = (p_1 - c_1 s_1) \left(\frac{p_2 - p_1}{\Delta s} - a \right) - A_0, \quad (7)$$

$$\Pi_2 = (p_2 - c_2 s_2) \left(b - \frac{p_2 - E p_1}{\Delta s} \right) - A_0. \quad (8)$$

Differentiating Π_1 with respect to p_1 we obtain $\left(\frac{p_2 - p_1}{\Delta s} - a \right) - \frac{(p_1 - c_1 s_1)}{\Delta s} = 0$, and the reaction function $p_1^m(p_2, c_1) = \frac{p_2 - a\Delta s + c_1 s_1}{2}$. The first order condition with respect to p_2 yields $\left(b - \frac{p_2 - E p_1^m(p_2, c_1)}{\Delta s} \right) -$

$\frac{(p_2 - c_2 s_2)}{\Delta s} = 0$, and so p_2^m satisfies:

$$\lambda \left[\left(b - \frac{p_2^m - p_1^m(p_2^m, c_3)}{\Delta s} \right) - \frac{(p_2^m - c_2 s_2)}{\Delta s} \right] + (1 - \lambda) \left[\left(b - \frac{p_2^m - p_1^m(p_2^m, c_4)}{\Delta s} \right) - \frac{(p_2^m - c_2 s_2)}{\Delta s} \right] \equiv 0. \quad (9)$$

Further, given that $E p_1^m(p_2, c_1) = \frac{p_2 - a \Delta s + (\lambda c_3 + (1 - \lambda) c_4) s_1}{2}$, p_2^m also satisfies:

$$(2b - a) \Delta s - 3p_2^m + (\lambda c_3 + (1 - \lambda) c_4) s_1 + 2c_2 s_2 \equiv 0. \quad (10)$$

On the other hand, when $(t_1 = a, t_2 = z)$, with probability $(1 - \lambda)$ firm 1 advertises the monopoly price, $p_1^t = v + a s_1 > p_1^m$, and with probability λ firm 1 competes for consumers in $[z, b]$, according to the reaction function $p_1^t(p_2, c_3) = \frac{p_2 - a \Delta s + c_3 s_1}{2}$. Therefore, we have that $p_1^t(p_2, c_3) \equiv p_1^m(p_2, c_3)$. Further, the first order condition which determines p_2^t is:

$$\lambda \left[\left(b - \frac{p_2 - p_1^t(p_2, c_3)}{\Delta s} \right) - \frac{(p_2 - c_2 s_2)}{\Delta s} \right] + (1 - \lambda)(b - z) = 0 \quad (11)$$

We now distinguish two cases:

a) If $z < \frac{p_2^m - p_1^m(p_2^m, c_4)}{\Delta s}$, then evaluating (11) in p_2^m , and using equation (9) we have that $\lambda \left[\left(b - \frac{p_2^m - p_1^t(p_2^m, c_3)}{\Delta s} \right) - \frac{(p_2^m - c_2 s_2)}{\Delta s} \right] + (1 - \lambda)(b - z) = (1 - \lambda) \left[(b - z) - \left(b - \frac{p_2^m - p_1^m(p_2^m, c_4)}{\Delta s} \right) + \frac{(p_2^m - c_2 s_2)}{\Delta s} \right] > 0$. Since $\left\{ \lambda \left[\left(b - \frac{p_2 - p_1^t(p_2, c_3)}{\Delta s} \right) - \frac{(p_2 - c_2 s_2)}{\Delta s} \right] + (1 - \lambda)(b - z) \right\}$ is a decreasing function of p_2 (second order conditions) it follows that $p_2^t > p_2^m$. Further, given that $p_1^t(p_2, c_3) \equiv p_1^m(p_2, c_3)$ and $\frac{\partial p_1^t(p_2, c_3)}{\partial p_2} > 0$, we have $p_1^t(p_2^t, c_3) > p_1^m(p_2^m, c_3)$. This completes the proof of the first part of (i).

b) Let us now consider $z > \frac{p_2^m - p_1^m(p_2^m, c_4)}{\Delta s}$. The first order condition with respect to p_2^t can also be expressed as $\frac{\lambda}{\Delta s}(b \Delta s - 2p_2 + p_1^t(p_2, c_3) + c_2 s_2) + (1 - \lambda)(b - z) = 0$. Substituting $p_1^t(p_2, c_3) = \frac{p_2 - a \Delta s + c_3 s_1}{2}$, we obtain $\frac{\lambda}{2 \Delta s}((2b - a) \Delta s - 3p_2 + c_3 s_1 + 2c_2 s_2) + (1 - \lambda)(b - z) = 0$. Next, we evaluate this expression in p_2^m , and use equation (10) to obtain $\frac{\lambda}{2 \Delta s}((2b - a) \Delta s - 3p_2^m + c_3 s_1 + 2c_2 s_2) + (1 - \lambda)(b - z) = \frac{\lambda}{2 \Delta s} [c_3 s_1 - (\lambda c_3 + (1 - \lambda) c_4) s_1] + (1 - \lambda)(b - z) = (1 - \lambda) \left[(b - z) - \frac{\lambda s_1 (c_4 - c_3)}{2 \Delta s} \right] > 0$ if and only if $s_2 > \frac{(c_4 - c_3) \lambda + 2(b - z)}{2(b - z)} s_1$. The concavity of $E \Pi_2$ implies that the condition $s_2 > \frac{(c_4 - c_3) \lambda + 2(b - z)}{2(b - z)} s_1$ is sufficient for $p_2^t > p_2^m$ which, in turn, implies $p_1^t(p_2^t, c_3) > p_1^m(p_2^m, c_3)$.

(ii) It is clear that targeting can cause a welfare loss only if $D_1^M = \text{Min} \left[(z - a), \frac{v + s_1(z - c_1)}{2 s_1} \right] = \frac{v + s_1(z - c_1)}{2 s_1}$. Therefore, we now solve the model for low values of v , i.e. $v < s_1(z + c_1 - 2a)$. In this case, when firm 1 monopolizes $[a, z]$, we have that $p_1^M = \frac{v + c_1 s_1 + z s_1}{2}$ and $\Pi_1^M = \frac{[v + s_1(z - c_1)]^2}{4 s_1} - A_0$, whereas if it competes for $[z, b]$, then the solution is identical to that in Proposition 2. Following the same steps as those described in the proof of Proposition 2, we have that a Bayesian Nash equilibrium can exist only if the following set of restrictions hold:

Restriction 3.1. $v < s_1(z + c_4 - 2a)$

$$\begin{aligned}
\text{Restriction 3.2. } & \frac{[p_2^t - a\Delta s - c_3 s_1]^2}{4\Delta s} > \frac{[v + s_1(z - c_3)]^2}{4s_1} \\
\text{Restriction 3.3. } & \frac{[p_2^t - a\Delta s - c_4 s_1]^2}{4\Delta s} < \frac{[v + s_1(z - c_4)]^2}{4s_1} \\
\text{Restriction 3.4. } & \frac{[p_2^t - z\Delta s - c_3 s_1]^2}{4\Delta s} - A_1 < \frac{[p_2^t - a\Delta s - c_3 s_1]^2}{4\Delta s} - A_0 \\
\text{Restriction 3.5. } & \frac{[p_2^t - z\Delta s - c_4 s_1]^2}{4\Delta s} - A_1 < \frac{[v + s_1(z - c_4)]^2}{4s_1} - A_0
\end{aligned}$$

Further, p_2^t , $E\Pi_2^t$ and $p_1^t(c_3)$ are identical to that obtained in Proposition 2.

Firm 2 has three possible deviations, which are identical to those described in Proposition 2, with the only difference that now $p_1^M = \frac{v + c_1 s_1 + z s_1}{2}$. Thus, an equilibrium exists if and only if the following set of restrictions hold: **Restriction 3.1., 3.2., 3.3., 3.4., 3.5., 2.6, 2.7, 2.8, 2.9., 2.11., 2.13., 2.14., 2.15., 2.16., 2.17. 2.18**, and further:

$$\begin{aligned}
\text{Restriction 3.6. } & E\Pi_2^d < E\Pi_2^t, \\
\text{with } E\Pi_2^d &= \frac{[3v(-1+\lambda) + s_1(8+4a(2-\lambda) - 3c_4(1-\lambda) - 5z(1-\lambda) - 4c_3\lambda) + 2s_2(-4+3c_2+z-2a(2-\lambda) - \lambda(z+c_2))]^2}{144\Delta s} - A_0. \\
\text{Restriction 3.7. } & \Pi_1^M = \frac{[v + z s_1 - c_1 s_1]^2}{4s_1} - A_0 > 0.
\end{aligned}$$

In order to check whether the parameter space imposed by these restrictions is empty, it is important to note that we are now studying a different type of equilibrium, namely, that in which $D_1^M < (z - a)$, and therefore, we must consider a new market scenario. It is straightforward to verify that the following market scenario: $[z, \lambda, a, v, s_1, s_2, c_3, c_4, c_2, A_0, A_1] = [0.4, 0.85, 0.1, 140, 119.8, 233, 0, 1, 0.7, 5, 3.5]$ satisfies all restrictions. Therefore, the equilibrium exists.

In order to compute how the transition from mass to targeted advertising affects welfare, we note that the monopolization of $[a, z]$ results in a loss of production of $[\theta^M - a]$, with $\theta^M = \frac{p_1^M - v}{s_1}$. Therefore, the change in social welfare induced by targeted advertising will be the difference between the social gain, related to the higher advertising efficiency induced by targeting, and the social loss derived from the quantity distortion provoked by the monopolist:

$$\Delta W = (A_0 - A_1) - \left[\int_a^{\theta^M} (v + x s_1) dx - \int_a^{\theta^M} c_4 s_1 dx \right].$$

We have computed this expression for the above market scenario, yielding that $\Delta W < 0$. This completes the proof of (ii).

Proof of Proposition 4:

The proof of this Proposition is similar to that in Proposition 2.

(a) • We begin by analyzing firm 1's optimal strategy. Under a continuous distribution of cost, when $t_1 = 0$ firm 1 must find it optimal to set $p_1^t(c_1) = p_1^M = v$ for all $c_1 > c_1^*$, and to set

$p_1^t(p_2, c_1) = \frac{p_2 + c_1 s_1}{2}$ for all $c_1 < c_1^*$. Therefore, c_1^* is a *cut off* value which is determined by a profit indifference condition between monopolizing $[a, z]$ or competing for $[z, b]$, that is to say, c_1^* must satisfy: $[(v - c_1^* s_1) z] = \frac{(p_2^t - c_1^* s_1)^2}{4\Delta s}$. Therefore, an equilibrium can exist only if:

Restriction 4.1. $\frac{[p_2^t - c_1 s_1]^2}{4\Delta s} > (v - c_1 s_1) z$ for all $c_1 < c_1^*$

Restriction 4.2. $\frac{[p_2^t - c_1 s_1]^2}{4\Delta s} < (v - c_1 s_1) z$ for all $c_1 > c_1^*$

Next, it is necessary to check that $t_1 = 0$ is indeed optimal. Firm 1 will not deviate from $t_1 = 0$ if and only if the following two restrictions hold:

Restriction 4.3. $\frac{[p_2^t - c_1 s_1 - z\Delta s]^2}{4\Delta s} - A_1 < \frac{[p_2^t - c_1 s_1]^2}{4\Delta s} - A_0$ for all $c_1 < c_1^*$

Restriction 4.4. $\frac{[p_2^t - c_1 s_1 - z\Delta s]^2}{4\Delta s} - A_1 < (v - c_1 s_1) z - A_0$ for all $c_1 > c_1^*$

• Regarding firm 2's strategy, we note that under a continuous distribution of cost, this firm only knows that with probability $(1 - c_1^*)$ the price charged by firm 1 will be:

$$E[p_1^t | c_1 > c_1^*] = \text{Max} \left[v, \frac{v + s_1(z + E[c_1 | c_1 > c_1^*])}{2} \right],$$

and so if $t_2 = z$, firm 2's demand will be: $D_2 = \text{Min} \left[1 - z, 1 - \frac{p_2 - E[p_1^t | c_1 > c_1^*]}{\Delta s} \right]$. Further, with probability c_1^* the price charged by firm 1 will be $E[p_1^t | c_1 < c_1^*] = \frac{p_2 + s_1 E[c_1 | c_1 < c_1^*]}{2}$, and firm 2's demand will be $D_2 = 1 - \frac{p_2 - E[p_1^t | c_1 < c_1^*]}{\Delta s}$, with $\frac{p_2 - E[p_1^t | c_1 < c_1^*]}{\Delta s} > z$. To focus the analysis on the most interesting case, we assume:

Restriction 4.5. $1 - \frac{p_2 - E[p_1^t | c_1 < c_1^*]}{\Delta s} > 0$,

Restriction 4.6. $1 - z < 1 - \frac{p_2 - E[p_1^t | c_1 > c_1^*]}{\Delta s}$

in such a way that, if $t_2 = z$, firm 2's problem is

$$\text{Max}_{p_2} E\Pi_2 = (p_2 - c_2 s_2) \left[c_1^* \left(1 - \frac{p_2 - E[p_1^t | c_1 < c_1^*]}{\Delta s} \right) + (1 - c_1^*) (1 - z) \right] - A_1. \quad (12)$$

Given that c_1 is described by a cumulative distribution function $F(c_1) = c_1$ with density $f(c_1) = 1$, we have that $E[c_1 | c_1 < c_1^*] = \int_0^{c_1^*} \frac{c_1 f(c_1)}{F(c_1^*)} dc_1 = \frac{c_1^*}{2}$. On the basis of this result, we obtain that firm 2 charges a price $p_2^t = \frac{4s_2[1 + c_2 c^* - (1 - c^*)z] + s_1[(c^*)^2 - 4 + 4z(1 - c^*)]}{6c^*}$ which, substituted in (12) yields an expected profit of $E\Pi_2^t$. Finally, $p_1^t(p_2^t, c_1) = \frac{p_2^t + c_1 s_1}{2}$, and substituting p_2^t in the *cut off* condition, we obtain that c_1^* is implicitly defined by $F(c_1^*, \alpha) = 0$, where:

$$F(c_1^*, \alpha) = (v - c_1^* s_1) z - \frac{\{4s_2[1 + c_2 c_1^* - (1 - c_1^*)z] + s_1[4z(1 - c_1^*) - 4 - 5(c_1^*)^2]\}^2}{24 c_1^* \Delta s}$$

and $\alpha = [z, v, s_1, s_2, c_2]$.

• Firm 2 has three possible deviations from $(t_2 = z, p_2^t)$. (i) One, it can set $(t_2 = z, p_2)$ such that $1 - \frac{p_2 - E[p_1^t | c_1 < c_1^*]}{\Delta s} < 0$. In this case, firm 2's problem is: $Max_{p_2} \Pi_2 = (p_2 - c_2 s_2) Min \left\{ 1 - z, 1 - \frac{p_2 - E[p_1^t | c_1 > c_1^*]}{\Delta s} \right\} (1 - \lambda) - A_1$, and this deviation will not be profitable if restrictions 2.8. and 2.9 hold. (ii) Two, if $t_2 = 0$, and $D'_2 = \lambda \left(b - \frac{p_2 - E[p_1^t | c_1 < c_1^*]}{\Delta s} \right) + (1 - \lambda) \left(b - \frac{p_2 - p_1^M}{\Delta s} \right)$, profit maximization yields

$$p_2^d = \frac{\Delta s + \left[c_1^* \frac{p_2^t + s_1 \frac{c_1^*}{2}}{2} + (1 - c_1^*) v \right] + c_2 s_2}{2}.$$

Substituting this price into the objective function yields $E\Pi_2^d$, and Firm 2 will not deviate if $E\Pi_2^d < E\Pi_2^t$. (iii) Three, if $t_2 = 0$, and $D'_2 = (1 - \lambda) \left(b - \frac{p_2 - p_1^M}{\Delta s} \right)$, then firm 2 will not deviate if restriction 2.11 hold.

Therefore, an equilibrium exists if and only if the following set of restrictions hold: **4.1., 4.2., 4.3., 4.4., 4.5., 4.6., 2.8., 2.9., 2.11.**, and further:

Restriction 4.7. $v \geq s_1(z + c_1)$, for all $c_1 > c_1^*$

Restriction 4.8. $E\Pi_2^d < E\Pi_2^t$

Restriction 4.9. $(v - c_1 s_1) z - A_0 > 0$, for all $c_1 > c_1^*$

Restriction 4.10. $\frac{[p_2^t - c_1 s_1]^2}{4\Delta s} - A_0 > 0$, for all $c_1 < c_1^*$

Restriction 4.11. $E\Pi_2^t > 0$

Restriction 4.12. $\Pi_1^m(c_1) > 0$, for all $c_1 \in [0, 1]$

Restriction 4.13. $E\Pi_2^m > 0$

Restriction 4.14. $U = v - p_1^m(c_1) > 0$, for all $c_1 \in [0, 1]$

We have computed the model for different market scenarios (see Table 1), and have found that, given $A_0 = 10, A_1 = (1 - z)A_0$, there exists a set of parameters, for example $[v, z, s_1, s_2, c_2] = [100, 0.3, 50, 150, 0.5]$, for which all the restrictions hold. This completes the proof of (a).

(b) We carry out this comparative static analysis by applying the Implicit Function Theorem to $F(c_1, \alpha) = 0$, which yields: $\frac{\partial c_1^*}{\partial \alpha} = -\frac{\frac{\partial F}{\partial \alpha}}{\frac{\partial F}{\partial c_1}}$. Given that $F(c_1, \alpha)$ is simply the profit difference for firm 1 between monopolizing $[a, z]$ and competing for $[z, b]$, a Bayesian equilibrium can exist only if it holds that $F(c_1, \alpha) < 0$, for all $c_1 < c_1^*$, and $F(c_1, \alpha) > 0$ for all $c_1 > c_1^*$. Therefore, in a neighborhood of c_1^* the function $F(c_1, \alpha)$ must be increasing in c_1 , i.e. $\frac{\partial F}{\partial c_1} > 0$, and so we have that $sign \left\{ \frac{\partial c_1^*}{\partial \alpha} \right\} = -sign \left\{ \frac{\partial F}{\partial \alpha} \right\}$. Next, we differentiate $F(c_1, \alpha)$, obtaining: $\frac{\partial F}{\partial z} = (v - c_1 s_1) + \frac{(p_2^t - c_1 s_1) \frac{1 - c_1}{c_1}}{3} > 0$, $\frac{\partial F}{\partial v} = z > 0$, $\frac{\partial F}{\partial c_2} = -\frac{(p_2^t - c_1 s_1) s_2}{3\Delta s} < 0$, $\frac{\partial F}{\partial A_j} = 0$, which completes the proof.

(c) This part of Proposition 4 is similar to Proposition 3, part (i), and so we only sketch the proof. Under mass advertising p_2^m satisfies: $1 - \frac{3p_2^m}{2\Delta s} + \frac{s_1 E[c_1]}{2\Delta s} + \frac{c_2 s_2}{\Delta s} = 0$, with $E[c_1] = \frac{1}{2}$, which implies $1 - \frac{3p_2^m}{2\Delta s} + \frac{c_2 s_2}{\Delta s} = -\frac{s_1}{4\Delta s}$. On the other hand, it is straightforward to check that $p_1^t(p_2) \equiv p_1^m(p_2)$, and that p_2^t is determined by: $c_1^* \left(1 - \frac{p_2 - E[p_1^t | c_1 < c_1^*]}{\Delta s}\right) + (1 - c_1^*)(1 - z) - c_1^* \frac{(p_2 - c_2 s_2)}{\Delta s} = 0$, with $E[p_1^t | c_1 < c_1^*] = \frac{p_2 + s_1 \frac{c_1^*}{2}}{2}$. Therefore, p_2^t satisfies: $c_1^* \left[1 - \frac{3p_2}{2\Delta s} + \frac{s_1 c_1^*}{4\Delta s} + \frac{c_2 s_2}{\Delta s}\right] + (1 - c_1^*)(1 - z) = 0$. If we evaluate this first order condition in p_2^m , we obtain $c_1^* \left[1 - \frac{3p_2^m}{2\Delta s} + \frac{s_1 c_1^*}{4\Delta s} + \frac{c_2 s_2}{\Delta s}\right] + (1 - c_1^*)(1 - z) = c_1^* \left[\frac{s_1 c_1^*}{4\Delta s} - \frac{s_1}{4\Delta s}\right] + (1 - c_1^*)(1 - z) = (1 - c_1^*) \left[(1 - z) - \frac{s_1 c_1^*}{4\Delta s}\right] > 0$ if $s_2 > \frac{5-4z}{4-4z} s_1$. Given the concavity of $E\Pi_2$ in p_2 (second order conditions), $s_2 > \frac{5-4z}{4-4z} s_1$ is sufficient for $p_2^t > p_2^m$. The rest of the proof is identical to that in Proposition 3.

Proof of Proposition 5:

First, we note that competition with mass advertising, i.e. $t_1 = t_2 = a$, yields $p_1^m = \frac{(b-2a)\Delta s + 2cs_1 + cs_2}{3}$, $p_2^m = \frac{(2b-a)\Delta s + cs_1 + 2cs_2}{3}$, $\theta^m = \frac{(a+b+c)}{3}$, $D_1^m = \frac{(b-2a)+c}{3}$, $D_2^m = \frac{(2b-a)-c}{3}$, $\Pi_1^m = \frac{\Delta s[(b-2a)+c]^2}{9} - A_0$, and $\Pi_2^m = \frac{\Delta s[c-(2b-a)]^2}{9} - A_0$. Next, we demonstrate that, given their rival's strategy, neither firm has an incentive to deviate from the equilibrium.

- We first analyze firm 1's optimal strategy. Given $(t_2 = \theta^m, p_2^t)$, firm 1 can monopolize $[a, z]$, according to the demand $D_1^M = \text{Min}[\theta^m - \frac{p_1 - v}{s_1}, \frac{b-2a+c}{3}]$ and, at the same time, this firm can compete for those consumers in $[z, b]$ by advertising a low price, $\hat{p}_1^t$, in the specialized media, i.e. $\hat{t}_1 = \theta^m$, in order to capture an additional demand $\hat{D}_1 = \text{Min}[\frac{p_2^t - \hat{p}_1}{\Delta s} - \theta^m, \frac{2b-a-c}{3}]$. To focus the analysis on the most interesting case, we assume that $\text{Min}[\theta^m - \frac{p_1 - v}{s_1}, \frac{b-2a+c}{3}] = \frac{b-2a+c}{3}$, which implies $v \geq \frac{(b-5a+4c)s_1}{3}$, and that $\text{Min}[\frac{p_2^t - \hat{p}_1}{\Delta s} - \theta^m, \frac{2b-a-c}{3}] = \frac{p_2^t - \hat{p}_1}{\Delta s} - \theta^m$. Under these conditions $p_1^t = v + as_1$ and $\hat{p}_1$ is determined by the maximization of: $(\hat{p}_1 - cs_1) \left[\frac{p_2^t - \hat{p}_1}{\Delta s} - \theta^m\right] - A_1$, which yields $\hat{p}_1^t = \frac{(b-2a)\Delta s + 8cs_1 + cs_2}{9}$. Given this result, it is straightforward to show that $\hat{p}_1^t < p_1^m < p_1^t$. Further, firm 1's profits are $\Pi_1^t = [v + (a - c)s_1] \left(\frac{b-2a+c}{3}\right) - A_0 + \frac{[b-2a+c]^2 \Delta s}{81} - A_1$, and so a Nash equilibrium can exist only if the price discrimination strategy is profitable, i.e. $\frac{[b-2a+c]^2 \Delta s}{81} - A_1 > 0$.

- Given $\left[(t_1 = a, p_1^t = v + as_1); \left(\hat{t}_1 = \theta^m, \hat{p}_1^t = \frac{(b-2a)\Delta s + 8cs_1 + cs_2}{9}\right)\right]$, if firm 2 sets $t_2 = \theta^m$, then p_2^t is obtained from $\text{Max}_{p_2} \Pi_2 = (p_2 - cs_2) \left(b - \frac{p_2 - \hat{p}_1^t}{\Delta s}\right) - A_1$, which yields $p_2^t = \frac{(5b-a)\Delta s + 4cs_1 + 5cs_2}{9} < p_2^m$, $D_2^t = \frac{5b-a-4c}{9}$, and $\Pi_2^t = \frac{[5b-a-4c]^2 \Delta s}{81} - A_1$. Starting from this solution, firm 2 has two possible deviations: (i) One, firm 2 can price discriminate by advertising p_2 in $t_2 = a$, in order to attract consumers in $[a, \theta^m]$ according to the demand $D_2 = \text{Min}[\theta^m - \frac{p_2 - (v + as_1)}{\Delta s}, \frac{b-2a+c}{3}]$, and $\hat{p}_2 < p_2$ in $\hat{t}_2 = \theta^m$ in order to attract consumers in $[\theta^m, b]$ according to the demand $\hat{D}_2 = \text{Min}[b -$

$\frac{\hat{p}_2 - \hat{p}_1^t}{\Delta s}, \frac{2b-a-c}{3}]$. Under this strategy, p_2 and $\hat{p}_2$ are determined by

$$\text{Max}_{p_2, \hat{p}_2} \Pi_2 = (\hat{p}_2 - cs_2) \left(b - \frac{\hat{p}_2 - \hat{p}_1^t}{\Delta s} \right) - A_1 + (p_2 - cs_2) \left(\theta^m - \frac{p_2 - (v + as_1)}{\Delta s} \right) - A_0$$

which yields $\hat{p}_2 = \frac{(5b-a)\Delta s + 4cs_1 + 5cs_2}{9} = p_2^t$, and $p_2 = \frac{(a+b)\Delta s + 3as_1 + 4cs_2 - cs_1 + 3v}{6}$. This type of deviation will be possible only if $\hat{p}_2 < p_2$. Therefore, in the case that $\hat{p}_2 > p_2$ (or, more generally, in the case that profits from price discrimination were negative, i.e. $\frac{[(a+b-2c)s_2 + 3v - (b-2a+c)s_1]^2}{36\Delta s} - A_0 < 0$), firm 2 will not find it optimal to deviate from $(t_2 = \theta^m, p_2^t)$ by price discriminating.

(ii) Two, firm 2 can advertise an uniform price p_2 in $t_2 = a$. In this case, the firm faces two demands: First, the demand from consumers in $[a, \theta^m]$ is given by $D_{21} = \text{Min}[\theta^m - \frac{p_2 - (v + as_1)}{\Delta s}, \frac{b-2a+c}{3}]$, with inverse demand $p_2 = \text{Max}[\theta^m \Delta s + v + as_1 - \Delta s D_{21}, v + as_2]$, and the demand from consumers in $[\theta^m, b]$ is given by $D_{22} = \text{Min}[b - \frac{p_2 - \hat{p}_1^t}{\Delta s}, \frac{2b-a-c}{3}]$, with inverse demand $p_2 = \text{Max}[\frac{2(5b-a)\Delta s + 8cs_1 + cs_2}{9} - \Delta s D_{22}, \frac{(4b+a+3c)\Delta s + 8cs_1 + cs_2}{9}]$. It is straightforward to show that (a) $\frac{(a+b)\Delta s + 3as_1 + 4cs_2 - cs_1 + 3v}{6} < \frac{(5b-a)\Delta s + 4cs_1 + 5cs_2}{9}$ implies $\frac{2(5b-a)\Delta s + 8cs_1 + cs_2}{9} > \theta^m \Delta s + v + as_1$, (b) that $p_1^m < v + as_1$ implies $\theta^m \Delta s + v + as_1 > \frac{(4b+a+3c)\Delta s + 8cs_1 + cs_2}{9}$ and, finally, (c) that $D_1^m > 0$ implies $\theta^m \Delta s + v + as_1 > v + as_2$. Accordingly, for the case in which $\frac{(4b+a+3c)\Delta s + 8cs_1 + cs_2}{9} > v + as_2$, firm 2 faces an aggregate demand:

$$D_2 = \begin{cases} 0 & \text{if } p_2 > \frac{2(5b-a)\Delta s + 8cs_1 + cs_2}{9} \\ b - \frac{p_2 - \frac{(b-2a)\Delta s + 8cs_1 + cs_2}{9}}{\Delta s} & \text{if } \frac{2(5b-a)\Delta s + 8cs_1 + cs_2}{9} > p_2 > \theta^m \Delta s + v + as_1 \\ \frac{13b+a+4c}{9} + \frac{v+as_1+cs_1}{\Delta s} - \frac{2p_2}{\Delta s} & \text{if } \theta^m \Delta s + v + as_1 > p_2 > \frac{(4b+a+3c)\Delta s + 8cs_1 + cs_2}{9} \\ b + \frac{as_1+v}{\Delta s} - \frac{p_2}{\Delta s} & \text{if } \frac{(4b+a+3c)\Delta s + 8cs_1 + cs_2}{9} > p_2 > v + as_2 \\ 1 & \text{if } p_2 < v + as_2 \end{cases}$$

whereas if $\frac{(4b+a+3c)\Delta s + 8cs_1 + cs_2}{9} < v + as_2$, the demand is

$$\tilde{D}_2 = \begin{cases} 0 & \text{if } p_2 > \frac{2(5b-a)\Delta s + 8cs_1 + cs_2}{9} \\ b - \frac{p_2 - \frac{(b-2a)\Delta s + 8cs_1 + cs_2}{9}}{\Delta s} & \text{if } \frac{2(5b-a)\Delta s + 8cs_1 + cs_2}{9} > p_2 > \theta^m \Delta s + as_1 \\ \frac{13b+a+4c}{9} + \frac{v+as_1+cs_1}{\Delta s} - \frac{2p_2}{\Delta s} & \text{if } \theta^m \Delta s + as_1 > p_2 > v + as_2 \\ \frac{13b-8a+3c}{9} + \frac{8cs_1+cs_2}{9\Delta s} - \frac{p_2}{\Delta s} & \text{if } v + as_2 > p_2 > \frac{(4b+a+3c)\Delta s + 8cs_1 + cs_2}{9} \\ 1 & \text{if } p_2 < \frac{(4b+a+3c)\Delta s + 8cs_1 + cs_2}{9} \end{cases}$$

Let us first consider that $\frac{(4b+a+3c)\Delta s + 8cs_1 + cs_2}{9} > v + as_2$. In this case the equilibrium cannot be related to the first section of D_2 with positive demand, given that this solution is dominated by $t_2 = \theta^m$. The optimal price corresponding to the following section of D_2 is given by the solution of:

$$\text{Max}_{p_2} \Pi_2 = (p_2 - cs_2) \left(\frac{13b+a+4c}{9} + \frac{v+as_1+cs_1}{\Delta s} - \frac{2p_2}{\Delta s} \right) - A_0,$$

which yields $p_2 = \frac{(13b+a)\Delta s + 9as_1 + 22cs_2 + 5cs_1 + 9v}{36}$, $\Pi_2 = \frac{[(13b+a-14c)s_2 - (13b-8a-5c)s_1 + 9v]^2}{648\Delta s} - A_0$. The equilibrium $(t_2 = a, p_2)$ can exist only if the following set of restriction hold:

$$\textbf{Restriction 5.1. } v \geq \frac{(b-5a+4c)s_1}{3}$$

$$\textbf{Restriction 5.2. } \frac{p_2^t - \tilde{p}_1^t}{\Delta s} - \theta^m < \frac{2b-a-c}{3}$$

$$\textbf{Restriction 5.3. } \frac{[b-2a+c]^2 \Delta s}{81} - A_1 > 0$$

$$\textbf{Restriction 5.4. } \frac{(a+b)\Delta s + 3as_1 + 4cs_2 - cs_1 + 3v}{6} < \frac{(5b-a)\Delta s + 4cs_1 + 5cs_2}{9}$$

$$\textbf{Restriction 5.5. } \theta^m \Delta s + v + as_1 > \frac{(13b+a)\Delta s + 9as_1 + 22cs_2 + 5cs_1 + 9v}{36} > \frac{(4b+a+3c)\Delta s + 8cs_1 + cs_2}{9}$$

Thus, the strategy $(t_2 = a, p_2)$ does not constitute a profitable deviation from $(t_2 = \theta^m, p_2^t)$ for the parameter space imposed by restrictions **5.1**, **5.2**, **5.3**, **5.4**, **5.5**, if the following restriction hold:

$$\textbf{Restriction 5.6. } \frac{[5b-a-4c]^2 \Delta s}{81} - A_1 > \frac{[(13b+a-14c)s_2 - (13b-8a-5c)s_1 + 9v]^2}{648\Delta s} - A_0$$

Let us now consider that $\frac{(4b+a+3c)\Delta s + 8cs_1 + cs_2}{9} < v + as_2$. In this case, the equilibrium price corresponding to the second section of $\tilde{D}_2$ is similar to that calculated for D_2 , whereas the equilibrium corresponding to the third section of $\tilde{D}_2$ is given by $p_2 = \frac{(13b-8a)\Delta s + 5cs_1 + 13cs_2}{18}$ and $\Pi_2 = \frac{[5c-13b+8a]^2 \Delta s}{324} - A_0$. This equilibrium can exist only if the following set of restriction hold: **5.1**, **5.2**, **5.3**, **5.4** and

$$\textbf{Restriction 5.7. } v + as_2 > \frac{(13b-8a)\Delta s + 5cs_1 + 13cs_2}{18} > \frac{(4b+a+3c)\Delta s + 8cs_1 + cs_2}{9}$$

Thus, the strategy $(t_2 = a, p_2)$ does not constitute a profitable deviation from $(t_2 = \theta^m, p_2^t)$ for the parameter space imposed by restrictions **5.1**, **5.2**, **5.3**, **5.4**, **5.7**, if the following restriction hold:

$$\textbf{Restriction 5.8. } \frac{[5b-a-4c]^2 \Delta s}{81} - A_1 > \frac{[5c-13b+8a]^2 \Delta s}{324} - A_0$$

We have computed the model for different market scenarios, and have found that, for the second section of D_2 and the third section of $\tilde{D}_2$, the parameter space which satisfies all these restrictions is not empty (see Tables 2 and 3, respectively).

v	100	100	100	150	150	150	200	200	200
s_2	150	150	150	150	200	200	200	250	250
s_1	50	50	50	50	50	50	50	50	50
c_2	0.5	0.7	0.6	0.7	0.7	0.9	0.7	0.7	0.9
c_1	0.3	0.9	0.9	0.3	0.6	0.9	0.3	0.6	0.9
z	0.3	0.5	0.6	0.3	0.5	0.7	0.3	0.5	0.7
c_1^*	0.95	0.84	0.48	0.8	0.69	0.5	0.86	0.64	0.44
θ^m	0.55	0.5	0.45	0.65	0.57	0.61	0.62	0.57	0.61
p_1^m	70	95	90	80	115.8	136.6	108.33	144.16	168.33
p_2^m	125	145	135	145	201.6	228.3	201.66	258.3	291.66
Π_1^m	20.25	15	10.25	32.25	39.11	46.01	48.07	55.17	66.05
Π_2^m	12.5	10	14.75	4	16.3	8.79	13.29	25.76	15.55
p_1^t	70.95			84.96	125.36		113.13	160.67	
p_1^M		100	100			150			200
p_2^t	126.9	149.72	159	154.9	220.7	254.1	211.27	291.34	337.72
Π_1^t	21.3			38.94	50.62		54.2	75.37	
Π_1^M		17.5	23			63.5			98.5
Π_2^t	15.86	17.36	23.63	7.99	24.4	19.23	17.64	35.32	30.81

Table 1. Equilibria with continuous production cost

$$(a = 0, A_0 = 10, A_1 = (1 + a - z)A_0)$$

v	a	s_1	s_2	c	A_0	A_1	p_1^m	p_2^m	Π_1^m	Π_2^m	p_1^t	$\hat{p}_1^t$	p_2^t	Π_1^t	Π_2^t
50	0	10	80	0.75	6	2.5	48.3	89.1	17.8	6.1	50	21.1	75.5	18.9	0.9
80	0	20	130	0.75	9	3.75	79.1	143.3	28.4	10	80	36.3	121.9	29.3	1.6
100	0	20	160	0.75	12	5	96.6	178.3	35.6	12.3	100	42.2	151.1	37.8	1.9
50	0	60	140	0.25	8	1.5	48.3	81.6	5.8	19.2	50	26.1	70.5	6.6	14.3
50	0	10	120	0.25	11	2	48.3	94.1	8	26.4	50	17.7	78.8	8.9	19.7
80	0	90	220	0.25	14	2.5	76.6	130.8	8.5	30.2	80	40.5	112.7	9.9	23.1
100	0	80	270	0.25	18	3.5	99.1	178.3	14.9	46.6	100	46.3	157.9	15.4	34
80	0.5	40	150	1	13	3	95	205	14.5	14.5	100	58.3	186.6	17	9.2
80	0.5	40	200	0.75	16	3	96.6	243.3	11.7	38.4	100	52.2	211.1	13.2	28.6

Table 2. Equilibria with price discrimination (D_2)

v	a	s_1	s_2	c	A_0	A_1	p_1^m	p_2^m	Π_1^m	Π_2^m	p_1^t	$\hat{p}_1^t$	p_2^t	Π_1^t	Π_2^t
50	0	20	100	0.25	12	1.5	38.3	71.6	1.8	15.2	50	16.1	6105	6.7	14.3
80	0	110	200	0.25	14	1.5	65	102.5	1.6	16.2	80	40	90	8.1	16.2
50	0	40	80	0.5	7	1	40	60	3	3	50	26.6	53.3	8.1	3.4
80	0	10	120	0.5	16	3	60	115	11.5	11.5	80	23.3	96.6	21.5	9.22
80	0	10	100	0.75	12	3	60	112.5	18.6	3.6	80	25	95	30.6	1.4
80	0.5	70	180	0.75	17	2	98.3	199.1	2	20.4	115	67.7	183.8	9.1	19.7
80	0.5	50	130	1	12	2	90	170	8	8	105	63.3	156.6	15.7	6.8
80	0.5	10	100	1.25	11	2	65	162.5	19.6	4.62	85	30	145	32.6	2.4

Table 3. Equilibria with price discrimination ($\tilde{D}_2$)

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